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SRAFFA'S PATH TO *PRODUCTION OF COMMODITIES BY MEANS OF COMMODITIES*. AN INTERPRETATION

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The present article provides a reconstruction of the path followed by Sraffa in the long journey (begun in the late 1920s) which brought him to his 1960 book. The starting point of Sraffa's research was the formulation of his cost-price equations, which surprisingly enough appear to have been derived neither from Ricardo's theory of value, nor from Marx's 'transformation of values into prices of production', but rather from Marx's reproduction schemes, published in volume II of *Capital*, to which Sraffa was almost certainly brought by his study of Marx's interpretation of Physiocratic theory in volume I of *Theories of Surplus Value*. The main device that Sraffa used in his attempts to bring to light the existence of solutions to his system and their properties, which basically meant building a consistent wage–profit–price relationship, was that of somehow cutting out the prices from this relationship. The paper shows that different routes were attempted by Sraffa to achieve this end, and that the main influences on these attempts were Ricardo and Marx, in particular Ricardo's corn-ratio theory of profits, and the related conception of a maximum rate of profits. In the course of his research, Sraffa was helped by some distinguished mathematicians, in particular Frank P. Ramsey and Abram S. Besicovitch. Of Ramsey's contribution there are important traces, but no more than traces, while ample records remain of the relationship between Sraffa and Besicovitch, and they will play an important part in the story told in this paper.

This work is based on a study of Sraffa's papers preserved at Trinity College, Cambridge. Its aim is to reconstruct as far as possible the path he followed in the research which led him to reach the remarkable results published in his 1960 *Production of*

* The text which follows is essentially based on a paper given at the conference for the centenary of Sraffa's birth organised in 1998 in Rome by *Fondazione Istituto Gramsci*, an Italian version of which is published in Pivetti (2000). It has materials added (relating to the discussion between Sraffa and Besicovitch on the existence and unicity of solutions) from a paper given in February 2003 to a conference at *Accademia Nazionale dei Lincei*, forthcoming in Italian in its proceedings. For very useful comments, I should like to thank Giorgio Gilibert, and also Geoff Harcourt, Luigi Pasinetti, Marco Piccioni and Massimo Pivetti. Thanks are also due to the Master and Fellows of Trinity College, Cambridge for the generous support given to my research, and to the staff of the Wren Library, for their help and forbearance over a period of several years. A special thank to Jonathan Smith, Modern Archivist at Trinity, for his constant and friendly help.

Commodities by Means of Commodities (*PCMC*).[1] More attention will be given in it to the path than to the results themselves, which of course anyone can see for himself in the book.

It goes without saying that a work like the present one can never aim at being definitive. Even irrespective of possible errors of interpretation, one cannot rule out that others may provide a partially or entirely different reconstruction of that path. The writing of a paper like this must necessarily be like the piecing together of a jigsaw puzzle, with the added difficulty that many of the pieces must be left out. It is therefore by no means impossible that using different pieces, a partly or entirely different picture may be put together.

Sraffa's book was published in May 1960.[2] In a version of the Preface, which did not get beyond proof stage,[3] he wrote that the work had started '**some thirty five years earlier**'.[4] This may suggest a stricter link with the research for his 1925 article[5] than with the 1926 one, which in fact is nowhere mentioned in the book.[6] This surmise is reinforced when we take into account a letter (of 6 October 1975) to C. P. Blitch, where Sraffa wrote that in the footnote at the beginning of his 1926 article he had written that it contained '**a summary of the conclusions**' of the 1925 article, in order '**to protect** [himself] **against giving the impression that** [he] **was selling the same article over again**'. But—he added—'**probably I overdid it. In economic theory the conclusions are sometimes less interesting than the route by which they are reached**'.[7] Incidentally, this remark upon the interest of the route followed by an author, perhaps allows us to assume that Sraffa would not be too horrified by the present attempt at reconstructing the route through which he reached the results published in 1960—though it is by no means claimed here that this route is more interesting than the conclusions themselves.

On the basis of the papers relating to the work on *PCMC* as they are presently available to us, I think it can be said that what is striking is not so much the length of time separating beginning from end, but rather the very long intervals within this period during which Sraffa appears to have left the work dormant. It started in the second half of the 1920s, and went on until the very beginning of the 1930s—I cannot recall any significant 1930s document with a date later than 1931, or perhaps 1932. From the very beginning of the 1930s we jump to the beginning of the 1940s, and find a substantial (both in the sense of quantity and of quality) amount of material dated 1941–1948 (mostly 1941–45). Then again we jump to the mid-fifties, and there is

[1] No discussion will be found in this study of problems concerning changes in techniques, or fixed capital and joint production—in other words, I shall only deal with problems relating to Part I of the book.

[2] On the 27th, but the official publication date was the 10th (see D3/12/111; all references to Sraffa's papers will be to the catalogue prepared for Trinity College by Jonathan Smith); the Italian edition was published on the 6th of June (Sraffa's diary, entries on their respective day).

[3] See proof copy of *PCMC*, item 3371 in Sraffa's library.

[4] For the sake of simplicity all quotations from Sraffa's papers will be in bold type.

[5] Sraffa (1925).

[6] Sraffa (1926). Also, in the following sentence he explicitly refers to the 1925 article.

[7] C 26.

much material dating from the period 1955–1958. What Sraffa himself regarded as the finishing touch was given on Wednesday 29th January 1958.[1]

What is also quite remarkable in the above series of dates is that they are an almost perfect complement of those marking the periods in which Sraffa was at work on the Ricardo edition. This in fact started in 1930, and probably went on for the whole decade: we know that the complete edition (as it stood before the great find of Ricardo MSS at Raheny,[2] and apart from the biographical volume, the index, and of course the editor's introductions)[3] reached proof stage in the summer of 1940.[4] In the following two or three years, however, work on Ricardo must have languished, so much so that in the early spring of 1943 Sraffa seemed on the point of withdrawing from the role of editor, following complaints from the Royal Economic Society about the enormous delay, which had meant among other things that Cambridge University Press had been holding 13 tons of type and 11 tons of paper for an unreasonably long period, and at a time of war shortages. The great Raheny find, a few months later (July 1943), was a double blessing in that it also gave Sraffa a reason to dodge the very latest deadline to which he had had to agree. Work on the newly discovered documents probably went on for some time, but it again reached a stalemate. In 1948 it restarted in earnest, when—again following complaints from the Royal Economic Society, when Keynes was no longer there to shield Sraffa—Maurice Dobb was called in to help with the edition (in particular with the writing of the introductions),[5] and ended in 1955 with the publication of volume X. (The index, as is well known, had still to wait a long time to be really dealt with.)

All this, of course, suggests that work on the Ricardo edition must have been for Sraffa alternative (perhaps even utterly irreconcilable) with work on his own research. At first sight this appears rather paradoxical. It is certainly true that the Ricardo edition required a lot of work on problems such as tracing manuscripts, checking and collating texts, collecting information on facts, on people, etc.; this work was time-consuming, and not easily compatible with an abstract piece of research as that on which Sraffa had embarked. But the more down-to-earth tasks must have been, to a great extent, accomplished in a few years. The main task laying ahead, that of writing the introductions—the task that, as Keynes suspected in 1943, was detaining Sraffa—one could see as subsidiary rather than adversary to the task of proceeding on the research for *PCMC*, if one judges from the finished product—which, as Sraffa himself wrote,[6] could to some extent be seen as a commentary on Ricardo. However, a point that comes out rather clearly from Sraffa's papers is that the path he followed in his research had no obvious

[1] Diary entry on that day: 'FINIS filled last gap in my work (Rent)'.

[2] On which see *Works*, I, ix (Sraffa, 1951–73, will be here quoted as *Works*, followed by the volume and page numbers).

[3] Virtually nothing seems to have survived of the early drafts of the introductions, but something may and should have existed, given that Keynes in August 1939 commented on what he described as the introduction to the *Essay on Profits* (D3/11/65). At this time probably only the introduction to volume I was not ready.

[4] *Works*, I, ix. The seven volumes of 1940 correspond to the first nine volumes of the published edition.

[5] On the collaboration between Sraffa and Dobb for the Ricardo edition see Pollitt (1990).

[6] See C/259.

connections with the problems in the theory of value as they had presented themselves to Ricardo. And only rather late (in the early 1940s) we find Sraffa considering in any depth Marx's problem of 'transforming values into prices of production'. Sraffa's point of depart appears to have been the price–cost relationship (in this we can see a link with his previous work on Marshall). The interest generated in Sraffa by the strictly materialistic conception of cost characteristic of the Physiocrats[1] (what Sraffa will refer to as the '**notion of "cost as the loaf of bread"**'), and the study of Marx's interpretation of Physiocratic theory in *Theories of Surplus Value*, brought Sraffa to volume II of *Capital*,[2] and to its reproduction schemes—and it is to Marx's reproduction schemes that Sraffa explicitly relates the formulation of the '**equations**', which will be discussed in this paper.

Of course it is not claimed here that the study of Ricardo that Sraffa made in the 1930s was irrelevant for the development of his thought. Ricardo (and more generally classical economic theory) was important, but its relevance was not so explicit, and not so direct. The problems Sraffa was tackling were not immediately the same problems—or at least for a long time they were not seen by him as the same problems—as Ricardo's. But Ricardo comes into the story at some crucial juncture, as will be shown in the present work.

What we find in Sraffa's papers regarding his theoretical research may, for convenience, be grouped in three different yet obviously related streams: the interpretation of classical theory, in particular of course the theory of value and distribution; the critique of marginalist theory; and the development of Sraffa's own theory, which for brevity's sake we may call his '**equations**'. The present work basically deals with the third of these streams, but the first will come in at some important points. The second, the critique of the marginalist theory of distribution, will not be prominent at all in what I shall say, but it must be remembered that of course it was an aspect of paramount importance to Sraffa from the very beginning, and from the very beginning some basic aspects of the faults of marginalist capital theory were already clear to him. It was of course when the main results of the '**equations**' were reached, that their full implications for the critique of the received doctrine were worked out, and this was in the early 1940s, in particular towards the end of 1943 to the beginning of 1944. (But hints of important aspects of the critique, e.g. the reswitching, can be found as early as 1942,[3] if not earlier.)

I. SRAFFA'S '**EQUATIONS**'

It is known that already in the autumn of 1927, shortly after his arrival in Cambridge, Sraffa started to show the initial parts of what was to become his book[4] to some of his

[1] Sraffa of course saw it clearly also in Petty.

[2] It is to be mentioned that the section on Physiocracy of *Theories of Surplus Value* reproduces part of volume II of *Capital*, to which it explicitly refers the reader.

[3] See D31/12/33/34, dated 21.4.42.

[4] Incidentally, we may record here that from very soon, i.e., at least from 1928, Sraffa envisaged himself as writing a *book*: this is remarkable, in the light of both Sraffa's reluctance to write, and the apparent paucity of results he had reached by that time.

new colleagues. Not only does Sraffa himself tell us in the Preface of *PCMC* that in 1928 he had shown Keynes '**a draft of the opening propositions**' of the book, but as early as the 26th of November 1927 he recorded in his diary '**K.**[eynes] **approves 1st equations**'—which in Sraffa's jargon meant equations without surplus ('**Second Equations**' meant equations with surplus but where labour doesn't appear explicitly, as in §§ 4–7 of *PCMC*; '**Third Equations**' meant equations with surplus, and with labour appearing explicitly). More or less at the same time he showed the equations to Pigou, who remarkably commented (in a letter to Sraffa of January 1928): 'Your equations seem to me to be capable of being subsumed as a special case of the general analysis. You in effect are simply supposing . . . constant returns'—though he characteristically added, 'I don't suppose for a moment that I've got your point'.[1] The texts Sraffa showed to Keynes and Pigou seem not to be extant.

Working on the '**equations**' implied a considerable shift of the focus of Sraffa's research from what he had been doing up to then. Until he had finished preparing his *Economic Journal* article (late in autumn 1926) his attention had of course to a great extent been absorbed by Marshall's theory and its critique. I don't know how deeply Sraffa had by then gone into the study of the classical economists and Marx. My impression is that at this time his knowledge of them (especially the classical economists) was not extensive, and much filtered through, and influenced by, other authors, notably Cannan, whose lectures at LSE he had attended in 1921–2, and who is often (and far from uncritically of course) referred to by Sraffa in his notes of the 1920s. My idea is that when he started preparing his lectures on '**Advanced theory of value**' for Cambridge[2] Sraffa began a more extensive study of the classical economists, and in particular of Marx. After all, even the expression 'Theory of Value' was associated more with the classics and Marx than with the marginalists, by whom the theory of value was often referred to as the 'theory of prices'. That Sraffa must at this stage have gone back to studying Marx, however, is also shown by his many notes from reading Marx (but also the classical economists, and others, like Pareto, Cassel, etc.), dated 1927 and 1928. As far as Marx is concerned, there are several quotations from *Theories of Surplus Value*, the so-called volume IV of *Capital*, which had been edited for publication by Kautsky in 1905–10. Sraffa, however, quotes *Theories of Surplus Value* from the French edition, the publication of which had only been completed in 1925. He, of course, owned (at least two) copies[3] of the 1905–10 original

[1] C 239/1. Another (much later, perhaps more interesting) comment by a prominent marginalist economist (and Pigou's successor), D.H. Robertson, was registered by Sraffa in two consecutive diary entries, of 17 and 18 April 1960: '**Dennis has read my ch. I, will read no more. "A wicked book, ought to be burnt"'. 'Dennis: not ashamed of yourself! an immoral book. Neo-Ricardian & Neo-Marxist**'. This is probably the earliest use on record of the term 'Neo-Ricardian' as referred to Sraffa. Many of Sraffa's economist friends and colleagues were shown the proofs, the first probably being Champernowne, who read a typed version of part I, and two appendices, as early as August 1957 (diary entry under 26 August). In a draft version of the preface (D3/12/99) Champernowne was thanked alongside Besicovitch, Ramsey and Watson.

[2] The preparation probably began early in 1927 (the first informal offer from Keynes of a lecturership at Cambridge seems to be of 25 January 1927, the official letter appointing him is dated 31 March 1927; the lectures were supposed to start the following October).

[3] Both however bought after the Second World War (in 1959 and 1967). None of them has any significant annotations. It is possible that he had owned other copies, because Sraffa often owned multiple copies of

German edition, but it is clear both from his notes and from his annotations within the book itself that his working copy was the French one—and indeed Sraffa usually referred to *Theories of Surplus Value* as *Histoire*, from its French title *Histoire des doctrines économiques*. This implies that his study of *Theories of Surplus Value* could not have started earlier than 1925. Also quotations of the other volumes of *Capital* in these 1927–8 notes are from the French edition. Although the first three volumes of *Capital* had been published in French much earlier than the *Histoire*, and Sraffa had almost certainly read *Capital* (at least volume I) earlier, it is safe to assume that probably in early 1927 he read (or rather at least in part re-read) Marx's *Capital*, with a view to preparing his lectures on '**Advanced theory of value**'.[1] Indeed there is a sketch (in a folder dated end of November 1927) of how he intended to start his lectures, which reads:

I shall begin by giving a short 'estratto' of what I believe is the essence of the classical theories of value, i.e., those which include W. Petty, Cantillon, Physiocrats, A. Smith, Ricardo & Marx.[2]

Although he only partially followed this project (the initial part of the lectures, on classical theory, being more historical than analytical), the fact that in 1927 he had this intention gives support to my contention that while preparing his lectures he must have (re-)read Marx and the classical economists.

It can be safely assumed that Sraffa's shift of emphasis, in 1926–27, from his critique of Marshall to his '**equations**', was mainly due to his (re-)reading of Marx. It is impossible to say with certainty whether it was the need to prepare lectures on the theory of value that pushed him to read Marx and the Classics, or the other way around, i.e., that he decided to give lectures on the theory of value because he had started a study of Marx and the classics. (The former appears more likely.)

Sraffa initially took a critical attitude towards Marx (and Smith and Ricardo). He thought that

A. Smith & Ricardo & Marx indeed began to corrupt the old idea of cost—from food to labour.[3]

footnote 3 continued from previous page

important books, which he used to sell or exchange with other books, but I doubt he would have sold his working copy (if not by mistake). It is of course possible that copies have gone missing. In Sraffa's library there is also a copy of the second (1872), the third (1883), and the fourth (1890) German editions of volume I of Marx's *Capital*, but none of them has any significant markings or annotations by him.

[1] In Sraffa's library there are annotated copies of volumes I–III of *Capital* in the French edition, which was published in 1872–75 (vol. I), and 1900–1 (vols II–III). There is a copy of the 1915 Italian edition of volume I of *Capital*, also containing many annotations, but from their contents I think they are likely to be of an earlier date than the annotations in the French edition. Throughout his work for *PCMC*, Sraffa's references are generally to the French edition of Marx's *Capital*. I think it can be said that in view of the need to prepare his lectures on the theory of value Sraffa went back to Marx, perhaps *in primis* to the *Histoire*, which had been published a couple of years before. From *Histoire* he returned to *Capital*, the first volume of which he had probably already read in the Italian edition, and read (perhaps for the first time) volumes II and III, from the copies of the French edition now at Trinity (no complete edition of volumes II and III was available in Italian until after the Second World War).

[2] D3/12/04/12.

[3] D3/12/04/02/i, in folder dated end of November 1927.

At the beginning his heroes seem to have been William Petty and the Physiocrats—'**it was only Petty & the Physiocrats who had the right notion of cost as "the loaf of bread"**'[1]—he writes.[2] But this, I believe, was only a very early and brief mood, perhaps a sort of remnant from the Marshall period—indeed, the great appeal of the '**notion of cost as "the loaf of bread"**', was for Sraffa that its firm materialistic approach was the farthest possible from what he regarded as the study of '**illusions**',[3] in which marginalist theory consisted. Early enough however his appreciation of Marx changed,[4] and in papers contained in the same folder as the passage on Marx's (and Smith's and Ricardo's) '**corruption**' of the old idea of cost, he could envisage that '**the ultimate result**' of his own work would be

a restatement of Marx, by substituting to his Hegelian metaphysics and terminology our own modern[5] metaphysics and terminology: by metaphysics here I mean, I suppose, the emotions that are associated with our terminology and frames (schemi mentali)—that is, what is absolutely necessary to make the theory living (lebendig), capable of assimilation and at all intelligible. If this is true, it is an exceptional example of how far a difference in metaphysics can make to us absolutely unintelligible an otherwise perfectly sound theory. This would be simply a translation of Marx into English, from the forms of hegelian metaphysics to the forms of Hume's metaphysics.[6]

His enthusiasm for Marx must have grown steadily:[7] he found it '**terrific**' that '**[i]n the middle of the 19th century a man** [i.e., Marx] **succeeds, either by accident or by superhuman effort, in getting again hold of the classical theory: he improves**

[1] Sraffa saw with favour Ricardo's early usage of the strange term 'price of wages', among the many expressions he used for this distributive variable. Sraffa actually regarded 'price of wages' as the 'proper name' (and criticised Marshall for criticising Ricardo) (D3/11/37). The reason was that this expression emphasised a distinction between wages (the real thing, the '**loaf of bread**'), and their price.

[2] D3/12/04/04, in folder dated end of November 1927.

[3] '**P**.[olitical] **E**.[conomy] **was a science of things, Economics is a science of illusions**' (D3/12/10/61).

[4] A student who read economics at Cambridge in 1932–5, many years later wrote in a letter to Sraffa: 'one of my favourite memories is of a meeting of Keynes' political economy club at which you defended Marx's Capital against a terrific onslaught' (C/291). This appears to give the lie to Austin Robinson's account of Sraffa as abstaining from speaking at Keynes's club, when Marx was discussed, and silently suffering Keynes's teasing him about whether 'there is anything in that chap Marx' (in Patinkin & Leith, 1977, p. 52).

[5] This word is inserted.

[6] D3/12/04/15.

[7] As a minor example of this enthusiasm, we may record that in 1932 he must have induced Keynes to borrow some books by Marx. Needless to say, Keynes was unimpressed, and returning the books wrote: 'I swear it absolutely beats me what you find in them, or what you expected me to find! I did not discover a single sentence of any conceivable interest to a rational human being. For next vacation you must give me a marked copy'. Sraffa must have been struck by Keynes's reaction, and two weeks later he transcribed part of Keynes's letter to R. Palme Dutt (the Communist head of the Labour Research Department, where Sraffa had worked for some time during his first stay in London in the early 1920s), adding some interesting remarks on the distance separating Marx not only from 'bourgeois intellectuals' like Keynes, but also from the working class, whose 'intellectual & literary food . . . is entirely provided ... by people of Keynes's mentality'. He therefore raised the problem of whether anything was being done to provide a 'mediation' which could make Marx intelligible to the working class (letter from Sraffa to R. Palme Dutt of 19 April 1932, National Museum of Labour History, Manchester, CP/IND/DUTT/06/02; I am indebted to Nerio Naldi for the knowledge of this letter).

it, and draws its practical consequences from it';[1] having conceived himself as simply '**translating Marx**', he started worrying about the structure of the book, to avoid the danger of finishing like Marx, who had proved incomprehensible to his contemporaries:

Impostazione del libro
L'unico sistema è di far la storia a ritroso e cioè: stato attuale dell'ec.; come vi si è giunti, mostrando la differenza e la superiorità delle vecchie teorie. Poi, esporre la teoria.
Se si va in ordine cronol., Petty, Fisiocr., Ric. Marx, Jevons, Marsh., bisogna farlo precedere da uno statement della mia teoria per spiegare dove si "drive at": il che significa esporre prima *tutta* la teoria. E allora c'è il pericolo di finire come Marx, che ha pubblicato prima il Cap., e poi non è riuscito a finire l'Histoire des Doctr. E il peggio si è che non è riuscito a farsi capire, senza la spiegaz. storica. Il mio scopo è: I esporre la storia, che è veramente l'essenziale II farmi capire: per il che si richiede che io vada dal noto all'ignoto, da Marshall a Marx, dalla disutilità al costo materiale.[2]

In view of this early preoccupation, it is curious that Sraffa—much more than Marx—in the end presented virtually nothing of the 'history' in his book (or even of its context), and—much more than Marx—ended up being incomprehensible to his contemporaries.

We now come to the equations themselves. It has already been mentioned that Sraffa speaks of first, second, and third equations. He starts of course with the first, representing a system without surplus. I shall adhere as much as possible to the chronological (which of course is also basically the logical) development of Sraffa's thought, and therefore deal first (and separately) with what we find in the papers up to the break of the early 1930s. This, however, creates a problem. In an early draft of the preface Sraffa wrote:

The matter of the first two chapters was completed by 1928, when it was submitted in a preliminary form to Mr J[ohn] **M**[aynard] **K**[eynes].[3]

On the other hand, it seems to me that it is only in the papers of the early 1940s that we find exactly the equations of *PCMC*. All the formulations of the equations we find in the earlier papers appear to have some problems which render them of difficult interpretation, and that no formulations of the '**Third equations**' (equations with surplus, and labour explictly mentioned) seems to be found before the 1940s.

[1] D3/12/04/17 (in folder dated end of November 1927).
[2] D3/12/11/55 (in scribbling pad dated November [1927, or perhaps 1928]). ('Structure of the book. The only way is to make history backwards, that is: present state of ec. how we got here, showing the difference and the superiority of the old theories. Then, expound the theory. If we proceed in chronol. order, Petty, Physiocr., Ric. Marx, Jevons, Marsh., it is necessary that it be preceded by a statement of my theory to explain the aim we drive at: which means to first expound *all* the theory. There is then the danger of ending up like Marx, who first published Cap., and then he did not manage to finish the Histoire des Doctr. And the worst is that he did not manage to make himself understood, without the historical explanation. My aim is: I expound the history, which is the really essential point. II make myself understood: for which it is required that I proceed from the known to the unknown, from Marshall to Marx, from disutility to material cost').
[3] D3/12/46/22.

II. THE '**EQUATIONS**': 1928–1931

Let us start with the '**first equations**', **i.e.,** the equations without surplus. Their early formulation is the following:

$$\mathbf{A} = \mathbf{a}_1 + \mathbf{b}_1 + \mathbf{c}_1$$
$$\mathbf{B} = \mathbf{a}_2 + \mathbf{b}_2 + \mathbf{c}_2$$
$$\mathbf{C} = \mathbf{a}_3 + \mathbf{b}_3 + \mathbf{c}_3$$

where A = Σa, B = Σb and C = Σc

On these equations he comments:

these are homogeneous linear equations. They have infinite sets of solutions, but the solutions of each set are proportional. These proportions[1] are univoche; these proportions we call[2] Absolute Values. They are purely numerical relations between the things A, B, . . . They are not necessarily the ratios in which exchange will actually take place . . . such actual ratios are also conditioned by such things as legal institutions, etc. which vary . . . and are 'arbitrary', i.e. irrelevant from our present point of view

and then writes a second set of equations:

$$\mathbf{aA} = \mathbf{a}_1\mathbf{A} + \mathbf{b}_1\mathbf{B} + \mathbf{c}_1\mathbf{C} \qquad \mathbf{a} = \Sigma\,\mathbf{a}\,\mathbf{?}$$
$$\mathbf{B} = \mathbf{a}_2\mathbf{A} + \mathbf{b}_2\mathbf{B} + \mathbf{c}_2\mathbf{C}$$
$$\mathbf{cC} = \mathbf{a}_3\mathbf{A} + \mathbf{b}_3\mathbf{B} + \mathbf{c}_3\mathbf{C}$$

which is the unit? A,B, . . . ?a?$\mathbf{a}_1$?[3]

These two sets of equations are puzzling. The first is very similar to Marx's equations of simple reproduction. Actually, Sraffa annotated his copy of (the 1900 French edition of) volume II of *Capital* to the effect that the equations of simple reproduction[4] are his '**1st equations**'.[5] For convenience, let us recall that Marx's equations are the following:

$$4000\ c + 1000\ v + 1000\ s = 6000 \text{ means of production} \qquad (2.1)$$
$$2000\ c + 500\ v + 500\ s = 3000 \text{ articles of consumption} \qquad (2.2)$$

Sraffa marks these equations pointing out that the sum of the first column is equal to the 6000 means of production produced, and that of the other two columns taken together is equal to the 3000 articles of consumption produced. It will be remembered that Marx explicitly assumes[6] that commodities exchange at their 'values' (embodied

[1] '**(ratios)**' inserted and then deleted.
[2] '**ratios of** ' inserted.
[3] D3/12/05/2-3 (in folder dated '**Winter 1927–28**').
[4] Page 444 of the 1900 Paris edition of volume II of *Capital* (this volume is n.3365 of Sraffa's library at Trinity). In the more easily available Kerr/Sonnenschein edition of 1907 this is at p. 459.
[5] In the MS index of relevant points on verso of p.591. Also, he writes in a document (dated 30 July 1942) '**Equations = Tableau Économique**' (D3/12/16/7).
[6] Page 454 of the Kerr edition of 1907.

labours), and he can accordingly write that '[t]he figures may indicate millions of francs, marks, pounds sterling, or dollars', or—we may add—simply embodied labours. In a sense, Marx's are not even equations strictly speaking (indeed, they contain no unknowns): they can be seen as accounting identities.

Sraffa instead speaks of his own relations as equations, with unknowns to which solutions have to be found. This has a meaning if we take the a_is, b_is, and c_is of the first set of equations as meaning the total value of each commodity input for the production of A, B, and C—the latter also interpreted as *the total values, not the quantities*, of the commodities produced. The second set of equations, on the other hand, can be seen as a more explicit formulation of the first set, if we now interpret A, B, and C as the prices of the three commodities; a, b, and c as the quantities produced; and the a_is, b_is, and c_is as the amounts of each commodity employed in the production of a, b, and c. After all, this is consistent with seeing the two sets of equations as deriving from Marx's reproduction schemes.

The above interpretation of the meaning of the variables in the equations has, however, at least two stumbling blocks. The first is that it implies that in the two sets of equations, the same symbols A, B, and C stand for different variables: value of total quantity produced of each commodity in the first set, price of each commodity in the second one. But this difficulty is not great, given that in a sense A, B, and C of the second set are as it were obtained by splitting the total value magnitudes A, B, and C of the first set in two magnitudes: a price (A, B, and C in their second meaning), and a quantity (a_i, b_i, and c_i).

The second obstacle is more worrying. In a paper dated June 1928, Sraffa summarised what appear to be observations by Frank Ramsey, and about the equations without surplus Sraffa's summary of Ramsey is the following:

Equations without surplus: each quantity must be expressed by *two* letters, one being the number of units, the other the unit of the commodity. Otherwise, if I use only one letter, this would stand for heterogeneous things and the sum would be meaningless.[1]

This appears to suggest that in the first set of equations above, the A_is, B_is, and C_is stand for *quantities*, not values (this is perhaps also reinforced by Sraffa's referring to them as '**things**'). But it may also be taken as suggesting just what Sraffa appears to do in writing the second set of equations—namely, splitting A, B, and C in two variables, a price and a quantity.

There is no doubt that the easiest way out of these difficulties would be to read the equations as simply making the mistake of summing heterogeneous things. But it would be very strange if Sraffa, who was already criticising the marginalists on the basis of the impossibility of summing heterogeneous quantities, should have been making the same mistake himself.

Perhaps a third, 'intermediate', but I am afraid not precise, interpretation of the above equations could be given. We know that Sraffa at this stage was still imbued with the conception that commodities—rather than to labour—should be reduced to

[1] D3/12/02/28.

their cost in terms of quantities of commodities, with a process analogous to that of reducing them to labour, and therefore prices and quantities of commodities were perhaps in some sense interchangeable (or confused?) in his mind; therefore *A*, *B*, and *C* could be seen by him as prices *and* quantities of commodities. If prices could be reduced to quantities of commodities, this in some sense involved seeing these various quantities as somehow homogeneous. Indeed, one of the (several) sheets in which equations of this type are written down (both for the case of production with surplus and without surplus) is entitled: '**Natural or physical value**'.[1] And in another, entitled '**Physical cost & value**' Sraffa writes:

When I say that the value of the product is 'determined' by the physical volume of commodities used up in its production, it should *not* be understood that it is determined by the value of those commodities. This would be a vicious circle, because—by what then is determined their value? Besides it would be wrong because the value of the product is equal to the value of the factors *plus* the surplus produced. What I say is simply that the numerical proportions between amount of factors and amount of product *is*, by definition, the absolute value of the product.[2] (November 1927 or 1928)

Indeed, for each product one could speak of an '**amount of factors**'—and reckon a numerical proportion between it and the '**amount of product**'—only assuming homogeneity between these factors.

I think, however, that this hermeneutic problem should not detain us too much. Whether it was Frank Ramsey or not to suggest it, Sraffa got to write the equations in the correct way, and, as already mentioned, an early formulation of the preface to *PCMC* suggested that they must have been fully spelled out by 1928.[3]

The proof that the '**First equations**' had a solution was relatively easy. As made clear in Sraffa's comment quoted above (and in others), it was based on the system being a system of linear homogeneous equations, which had infinite sets of proportional roots, one of the equations being not independent of the others. It seems to me that there is no explicit proof that the roots were all positive, nor indeed of the question itself. One might think that Sraffa knew that when there is no surplus, and therefore no profits, prices are equal to Marx's 'values'—which of course implies that they must be all positive. But it is not clear how Sraffa would have known this. He still did not accept (at least not fully) the reduction to labour, and (in a document written probably in 1942) criticised the marginalists for regarding it as trivial that if there are no profits, the labour theory of value holds.[4] Nor does he appear at this stage to have developed the conception of a 'Sub-system'. However, he seems to have been aware of the equality of labour embodied and labour commanded when there are no profits,[5]

[1] D3/12/11/75.

[2] D3/12/11/101.

[3] It is, however, possible that the early formulation of the preface was dropped because it imputed too much to 1928: '**The matter of the first two chapters was completed**' suggests a state of finality by no means implied by '**a draft of the opening propositions**'—the published version of the preface.

[4] D3/12/15/9/2.

[5] See D3/12/16/13/3 (of August 1942).

and this might at this stage have led him to see that at zero profits, prices must be equal to labour embodied.[1]

The introduction of surplus must have given Sraffa a headache. The equations from which he appears to start are of the following type:

$$\begin{aligned} \mathbf{10A} &= \mathbf{3A} + \mathbf{2B} + \mathbf{6C} \\ \mathbf{15B} &= \mathbf{1A} + \mathbf{3B} + \mathbf{2C} \\ \mathbf{13C} &= \mathbf{2A} + \mathbf{4B} + \mathbf{1C} \\ \mathbf{S} &= \mathbf{4A} + \mathbf{6B} + \mathbf{4C} \end{aligned}$$

and he comments:

The system is *contradictory*, because sum of terms on the left is greater than sum of terms on the right unless S is zero. But if S is zero, then the last equation is selfcontradictory, because the sum of its terms on the right is significant.[2] (Winter 1927)

The rationale of writing this kind of equations appears to be in the following remark:

consider[o] **i vari surplus come merci diverse (supponendo cioè che appena sorge il surplus, si continua a produrre la stessa quantità di 'grano' di prima, e le risorse residue vengon tutte dedicate a produrre gioielli e altre cose 'improduttive').**[3] (November 1927 or 1928)

The logic of this kind of equations is not fully clear, nor is it clear how Sraffa should have been able to apparently derive from them the following conclusion (which actually foreshadows an important point of the distinction between basics and non basics):

$$\begin{aligned} \mathbf{A} &= (\mathbf{a} + \mathbf{b} + \ldots)\mathbf{r} \\ \mathbf{B} &= (\mathbf{a_1} + \mathbf{b_1} + \ldots)\mathbf{r} \\ &\ldots\ldots \\ \mathbf{S} &= (\mathbf{a_s} + \mathbf{b_s} + \ldots)\mathbf{r} \end{aligned}$$

'factors' used in A, B . . . are 'productive' because, if we assume that, say, b_2 can be reduced to ½ while *all* products remain unchanged, *r* increases. i.e. such industries produce *a surplus*, i.e. *r varies* with them. But if b_s is reduced to ½, *r* remains unchanged (well, but then what do you do with the superabundant B?).
Better put it thus: if physical amount of S changes nothing happens: if physical amount of A, or B . . . increases (whether or not? superabundance is used for S) *r*

[1] Another problem is that the difficulties arising from fixed capital had started to bother Sraffa rather early, and rather early—probably through Wicksell—he had seen the difficulties for the conception of the quantity of labour embodied in a commodity, which could arise from the existence of fixed capital and joint production (these difficulties are spelled out by Wicksell in his review of Åkerman: Wicksell 1923).

[2] D3/12/10/67. He seems to be still struggling with this problem in November 1928: see D3/12/11/17. Here however among the possible solutions he also has that of '**add**[ing] **as an unknown the rate of interest**', a passage he later marked as important.

[3] D3/12/11/87 ('I consider the different surpluses as different commodities (i.e. assuming that as soon as a surplus arises, the same quantity of "corn" is produced, and the remaining resources are all employed to produce jewels and other "unproductive" things').

increases. Therefore those industries are productive which produce a surplus (i.e. *r* varies with their efficiency).
Those are improductive [sic] **which compose the *things* composing *the* surplus of society, i.e. luxuries.**[1] (May–July 1928)

It should be mentioned that Sraffa at this initial stage does not reckon profits on commodities used in their own production: for some time (probably not beyond 1927–28) he regarded '**seed**' (i.e., each commodity used in its own production) as a sort of fixed capital,[2] and thought that on fixed capital (or at least on the part of it which was not consumed in the year) no profit had to be paid.[3] The difficulty he had at this stage with profits on fixed capital is partly illustrated by the following remarks, made in a paper entitled '**Interest on fixed capital**':

The fixed capital may be considered as part of the product: but it must have the same value on the right and left of the equations. How can it then, when it appears as product, include profit? The other products are of the same value on the right and left, but they include profit because, as products, they are in greater quantities. Could not also fixed capital increase in quantity? And then not all (and not all equally) products are increased in amount.[4] (May–July 1928)

Indeed, at this stage Sraffa regarded Malthus's (i.e., Torrens's) way of treating fixed capital as a fallacy, into which also Marx had fallen. Be that as it may, he certainly at some stage in 1928 thought that profit on fixed capital had only to be paid when there was accumulation.[5]

The nearest approximation to the formulation of the '**Second equations**' as we know them in *PCMC* seems to be the following, which could actually be as early as 1927:

$$\mathbf{v_a}\,\Sigma\,\mathbf{a} = (\mathbf{v_1}\,\mathbf{a_0} + \mathbf{b_1} + \mathbf{v_c}\,\mathbf{c_1})\,(\mathbf{1} + \mathbf{r})$$

$$\Sigma\,\mathbf{b} = (\mathbf{v_1}\,\mathbf{a_2} + \mathbf{b_0} + \mathbf{v_c}\,\mathbf{c_2})\,(\mathbf{1} + \mathbf{r})$$

$$\mathbf{v_c}\,\Sigma\,\mathbf{c} = (\mathbf{v_1}\,\mathbf{a_3} + \mathbf{b_3} + \mathbf{v_c}\,\mathbf{c_0})\,(\mathbf{1} + \mathbf{r})$$ [6]

[1] D3/12/09/45.

[2] Sraffa actually quotes Smith to this effect: 'The whole value of the seed . . . is properly a fixed capital' (Smith, 1789, I, 263). Smith's distinction between fixed and circulating capital at this juncture is that capital from which profit is obtained by parting from it is regarded as circulating, while capital from which profit is obtained by keeping it is fixed (e.g. cattle which is bought to be resold is circulating capital, cattle which is bought to obtain wool or milk from it is fixed capital: *ibid.*). Sraffa also notices that Malthus (1827, p.237–8) uses Smith's distinction between fixed and circulating capital.

[3] D3/12/10/63 (in scribbling pad dated Winter 1927). Sraffa also quotes the entry 'Interest & usury' in Palgrave's *Dictionary* as stating that theologians distinguished between interest on circulating capital as legitimate, and interest on fixed capital as exploitation or dead hand.

[4] D3/12/09/21.

[5] See D3/12/09/11.

[6] D3/12/07/58. The suffix 0 in each of the three equations denotes the quantity of a commodity that enters its own production—the 'seed'. It must also be noted that the suffixes are here given correctly, but there are some slips in the actual document, where the price of c for instance is sometimes called v sometimes v_2. Also, the equations appear to have been in the first place written without mention of prices, which are inserted.

(It is perhaps also worth mentioning that Sraffa had contemplated the possibility of 'closing' his system by giving the proportions in which workers and capitalists consume the commodities they consume;[1] the idea, however, does not reappear afterwards.)

Before the 1940s there seems to be no discussion of a proof that the equations with surplus had a solution, let alone that it was unique and positive. It is indeed possible that, when he suspended his work on the equations to start the Ricardo edition in the early 1930s, Sraffa thought that it was very difficult, if not impossible, to find a solution for the equations with a surplus, at least if the rate of wages was taken as the independent variable. In a slip of paper inserted among other papers mostly dated 1931 (and itself of possibly a later date), he writes:

Given w, is solution possible? r at the power n, . . . If r unknown it cannot be found even if values and w are known.[2]

It must, however, be mentioned that already in 1928 Frank Ramsey had told Sraffa that approximate solutions could probably be found for his systems of equations, with any number of equations, and that **'It can probably be proved that, whatever the number of equations, only *one* set of solutions is significant'**.[3] There is no clue as to the basis of Ramsey's statements, made at such an early stage. It is plausible that Ramsey gave Sraffa some reasons for his assertions, and that Sraffa may have been helped by them in his search for a way of 'opening up' his systems of equations.

III. SRAFFA'S '**EQUATIONS**' IN THE 1940S: THE CRUCIAL DEVELOPMENTS

It is well known that Sraffa spent three months in a British concentration camp—from the 4th July to the 9th October 1940. It is less well known that he employed part of this time re-reading volume I of Marx's *Capital*: in his library at Trinity there is a heavily annotated copy of the 1938 facsimile reprint of the 1889 English edition, dated by Sraffa on the front free endpaper '**Metropole Internment Camp Isle of Man Sept. 1940**'.[4] Back in Cambridge, however, he had an unusually heavy teaching load, due to the absence of many of his colleagues on war jobs. However, it appears that in 1942 Sraffa resumed work on his book; the period from 1942 to 1944 (or 1945) was that of the crucial breakthroughs, which is illustrated below. In this period, Sraffa probably sought, and certainly obtained, the help of the mathematician A. S. Besicovitch, a fellow of Trinity College (where Sraffa himself had moved, having obtained a fellowship, in 1939). This help Sraffa will describe as '**invaluable**' in the preface of *PCMC*.

It could be said that the problem which Sraffa envisaged was basically that of building a consistent wage–profit–price relationship. Prices enter this relationship as a dis-

[1] See D3/12/13/14 (in folder dated Winter 1927–1928).
[2] D3/12/07/167.
[3] D3/12/02/28, dated 26 June 1928.
[4] In Sraffa's library there is another book that bears a similar inscription: it is a Penguin edition of Tolstoy's *War and Peace*.

turbing element: it is because prices depend on distribution that the wage-profit relationship cannot be a simple (linear) one. The study of how the 'cake' is divided into the slices distributed to the 'factors of production' is complicated by the fact that when these slices vary the *value* of the cake to be divided varies.

In the 1940s, Sraffa had clearly in front of him the solutions given by Ricardo, and the one given by Marx; he certainly was aware that they were not fully satisfactory, but he almost certainly saw that they showed a possible way to follow: that of 'cutting out' the prices. In fact a point which comes out rather clearly from Sraffa's attempts at making his problem manageable is that he often tries to reduce the problem to linearity, that is basically to cut out prices.[1] This sometimes led him into making mistakes, as will be shown below, but in the end it bore fruit.

It can be said that the first shortcut envisaged by Sraffa was suggested by Ricardo. In the papers of 1942 we find in a prominent place what Sraffa will later call the 'corn-ratio theory of profits', i.e., the determination of the rate of profits as a ratio between two physical quantities of the same commodity (corn in Ricardo).[2] In a paper entitled '**Equations with profits Ricardo's case**' Sraffa writes:

If, in the production of corn, the only article used is corn (for seed & sustenance of workers) then the equation for the corn industry, by itself, determines the rate of profits for *all* industries. This rate is, in this case, independent of the values.
This is a result of the same commodity, alone, appearing on both sides of the corn equation, so that the rate of profits can be immediately calculated, independently of the other equations. Another condition, of course, is that corn should be used in the production of *all* other commodities.
N.B. What are the limits within which the above case is possible without making the system of equations contradictory?[3] (16.2.1942)

It is possible that at this juncture Sraffa sought to generalise Ricardo's corn–corn solution to determine the rate of profits (which he will later do by means of the Standard Commodity). We find in fact a document of September 1942, in Besicovitch's hand,[4] where Besicovitch shows that the system of equations for the production of k individual commodities can be represented as a system where k composite commodi-

[1] When the rate of profits is expressed as a ratio between physically homogeneous quantities (as in Ricardo's 'corn' or Sraffa's 'Standard Commodity') the wage-profit relationship is linear if wages are paid *post-factum* (at the end of the year). If they are paid at the beginning of the year, notwithstanding the fact that prices are cut out of the *w-r* relationship, this relationship is not linear, but a hyperbole (see Pasinetti, 1975, p.161; 1977, p.132).

[2] It would appear that by early 1942 Sraffa was aware of Ricardo's corn-ratio theory of profits. As is well known, in *PCMC*'s 'References to the Literature' Sraffa states that 'it was only when the Standard system and the distinction between basics and non-basics had emerged in the course of the present investigation that the [corn-ratio] interpretation of Ricardo's theory suggested itself as a natural consequence' (p.93). Given that, as will be shown in the text, the Standard system was only devised in 1944, it would appear that the *a posteriori* reconstruction given by Sraffa in 1960 is contradicted by his papers. We cannot enter into a discussion of this problem here.

[3] D3/12/16/39.

[4] This is one of the earliest documents in Besicovitch's hand found in Sraffa's papers; for several reasons I am inclined to think that this demonstration was formulated by Besicovitch at Sraffa's request, rather than autonomously, as will instead be probably the case for the example which will be discussed below.

ties are produced, every one of which produced only by means of itself (i.e., by means of a set of individual commodities taken in the same proportions in which they appear as product) and labour. Crucially, however, Besicovitch remarks that 'some of the simple commodities may enter into the compound commodities as *negative* quantities'.[1] At this stage this constituted an important obstacle for Sraffa on the way suggested by Ricardo's 'corn', and probably caused him to abandon this route for a while.[2]

From Ricardo's corn–corn reasoning, however, came to Sraffa the suggestion of another route to follow. Prominent among his reflections in 1942 is the concept of a maximum rate of profits. In a paper entitled '**Second equations**' (and, added later, '[**Profits of the farmer determine all other profits. Ricardo**]'),[3] he develops this point in the following way:

a) in some industries the product itself ('seed') appears among the materials used up. The ratio of the amount of it used in its own production to the amount produced sets a maximum limit to the rate of profits. That, among such industries, which has the lowest maximum sets the *maximum limit* of the rate for all industries

b) If in one industry (e. g. wheat) its own product is used as the only material, the ratio of material to product gives directly the rate of interest; and if this product is used in each of the other industries as material, the rate of profits of the wheat industry *determines* the rate of all others. Note: it is implicit in the conditions that there can be only *one* such industry [Note: this is Ricardo's case] (a and b are in general compatible). There is a contradiction between a and b if the maximum set for (by) 'other' industries is lower than the rate determined in the 'wheat' industry.[4] (8.7.1942)

He then notices that this maximum rate of profits must be equal to the ratio of the value of the product to the value of capital:

***r* has a maximum for society, that is ratio of total product to capital.**[5] (5.8.1942)

More or less at this time, he starts to write about the '**hypothesis**', or the necessity, that the ratio between the value of the product and the value of constant (i.e., non-wage) capital should not change with changes in distribution. At first sight this appears really puzzling: it seems a most un-Sraffian hypothesis to make. Moreover, in the many places where it reappears time and again—so much so that Sraffa starts to call it '**My**

[1] D3/12/14-5.

[2] As a matter of fact sometimes Sraffa mentions **'Besicovitch's compound commodities'** as a stumbling block: see for instance the document D3/12/36/81, dated 1 February 1944, i.e. almost two years after Besicovitch's demonstration.

[3] It is well known that in 'References to the Literature' of *PCMC*, Sraffa mentions Marx rather than Ricardo as the source for the notion of a maximum rate of profits. The fact is however that the conception is not prominent in Marx, and the evidence Sraffa quotes from him is rather slim. The conception really seems a deduction by Sraffa from elements of both Ricardo and Marx. The reasons why Sraffa emphasised its derivation from Marx rather than Ricardo I think were basically two: (a)Once Ricardo abandoned the corn-ratio theory, and came to rely on a reduction to labour in a finite number of stages, he lost the idea of a maximum rate of profits; (b)Marx's formulation of the rate of profits as the ratio $S/(C + V)$ was important for Sraffa's reasoning about the maximum rate of profits, as shown in the text.

[4] D3/12/16/37.

[5] D3/12/16/9.

Hypothesis' or '**The Hypothesis**'—he appears not to take it as a provisional and unwarranted assumption, but argues at length, in various ways, that this constancy, or rather the conditions for its validity (equal 'organic composition' for product and capital) actually hold in general. In fact, he goes so far as twisting Marx's reason for calling non-wage capital 'constant' capital, writing that the means of production are

the constant part of capital, as their quantity does not change when the proportions in which the rest of the product is distributed between wages & profits are changed.[1] (31.8.1942)

Sraffa appears to think that, if the '**Hypothesis**' held, it was possible to proceed as follows: making $r = 0$ determine prices (which would be equal to labours embodied); reckoning product and capital at these prices,[2] determine the 'true' r; by means of the 'true' r determine the 'true' prices. Basically, Sraffa thought that by means of the '**hypothesis**' he could do what Marx had done in his 'transformation', i.e., use *given* prices (equal to embodied labours, i.e., the prices determined for $r = 0$) in order to determine the rate of profits, and in turn use this rate of profits to determine the prices[3]. The validity of the '**hypothesis**' would have warranted the validity of this way of proceeding. At this stage Sraffa even thought that he had rehabilitated '**Old Moor**' (i.e., Karl Marx).

Rather soon, however, (in the same month of August 1942, or shortly afterwards) Sraffa realised that this '**model**' did not really work. Actually, we find him writing about a '**Disastro del modello**', clearly referring to problems about the assumed constancy in the value of capital in terms of product.[4] For quite some time, however, he goes on trying to explore whether such disaster could be remedied;[5] and proceeds on a double line as it were, discussing problems on both the assumption that the '**hypothesis**' holds, and that it does not.

It seems interesting to notice that what Sraffa deemed disastrous if the hypothesis did not hold, was that the w-r relationship was no longer linear. It is also interesting to see what he writes on how to proceed, after the disaster:

The non-linearity of wr in cases where our hypothesis fails even if only a little[6] must be studied: it now seems disastrous. But it may contain just the remedy wanted to re-establish, in effect, our hypothesis & the whole theory.[7]

We shall see that in a sense this is just what he did, and the method proved fruitful.

Sraffa's idea that the value ratio between product and constant capital should not vary with changes in distribution derived from the conception of a maximum rate of

[1] D3/12/19/1.

[2] Thanks to the '**hypothesis**' the ratio between value of the product and value of the capital would be the same at all levels of r.

[3] For an example, see D3/12/19/iii, of 31 August 1942; also D3/12/21/61 (itself not dated but kept in a folder dated September 1942).

[4] See for instance D3/12/20/5.

[5] He writes for instance: '**1) Circ[olante]. Vedere se *da solo*, col suo prodotto, rimane costante in termini del prodotto**'(D3/12/20/5; n.d. but in folder d. August 1942).

[6] last five words inserted later.

[7] D3/12/33/64, dated 30 March 1943.

profits. Given that the maximum rate of profits did not change with changes in distribution (Sraffa had clearly seen that it depended solely upon conditions of production), and that the maximum rate of profits must be equal to the ratio of net product to constant capital, the latter ratio ought not to vary with changes in distribution.[1] Sraffa's deduction is of course wrong, but in the end it proved useful, as we shall see.

On the basis of his 'hypothesis', Sraffa makes the following remarks:

We add up all the *n* equations and obtain a general equation

$$(\mathbf{A_a}+\mathbf{A_b}+\ldots+\mathbf{A_n}+\mathbf{p_bB_a}+\ldots+\mathbf{p_nN_n})(1+\mathbf{r})+(\mathbf{wL_a}+\mathbf{wL_{b+}}\ldots+\mathbf{wL_n})(1+\mathbf{r})$$
$$=\mathbf{A_t}+\mathbf{p_bB_t}+\ldots+\mathbf{p_nN_t}^2$$

We know that the value of the aggregate of commodities within the first bracket (constant capital) bears a constant ratio to the value of the commodities on the right hand side (social product); constant, that is to say, in relation to the variations of the particular prices as wages and profits change.
We can therefore replace both aggregates with two quantities of an imaginary composite commodity, made up of the same commodities in the same proportions as the social product: we shall call it the Average Commodity [or General?] and denote it by the letter C. The *quantity* of C used in production (C_c) will have the same ratio to the *quantity* produced (C_t) as the *value* of the aggregate of commodities composing social constant capital has to the *value* of the aggregate composing the social product. And the quantity of labour used will be equal to the total quantity of labour. We thus get the *general* equation

$$(\mathbf{C_c}+\mathbf{w}\,\mathbf{L_c})\,(1+\mathbf{r})=\mathbf{C_t}$$

. . .

the prices p_a, p_b, ... p_n as well as wages are now expressed in terms of the average commodity C.[3]

From this '**general equation**' Sraffa of course sees that

the maximum to which *r* can rise (i.e., when w=0) is

$$\mathbf{r_{max}}=\frac{C_t-C_c}{\mathbf{C_c}}=\frac{C_t}{\mathbf{C_c}}-1$$

therefore

$$(1+\mathbf{r_{max}})=\mathbf{C_t}/\mathbf{C_c}.$$[4]

The same concepts are repeated time and again in other papers of the same period. In a paper of October 1942 Sraffa sees that assuming (as he was doing) that the ratio

[1] There might have been another element in the reasoning: if *R* is given, and is the ratio between value of net product and value of constant capital, and does not change with changes in *w* and *r*, then it must be the same also when $r = 0$, and commodities exchange according to labour embodied, so the ratio between value of product and value of constant capital must be equal to ratios of labour embodied.

[2] The A_t, $B_{t,\ldots,}N_t$ are, of course, the total quantities produced.

[3] Against the whole paragraph starting '**We can therefore**' a vertical mark of dissent and the following words: '**Better say: value of C_c equal to aggr. const., value of C_t equal to value of social product**'.

[4] D3/12/24/25-6 (dated 7.10.1942).

between the value of the product and the value of non-wage capital is invariant with respect to distribution, is equivalent to assuming that they are the same commodity.[1] This brings him a few months later to the following argument: given that

$$C = (C_c + L\,w)(1+r)$$

$$C = C_c\,(1 + R) \qquad [\text{if } w = 0,\ r = r_{max} = R]^2$$

$$C = C_c + L \qquad [\text{if } r = 0,\ w = 1]^3$$

one can write:

$$L = C_c\,R$$

$$C_c = L/R$$

Therefore

$$(L/R + Lw)\,(1 + r) = L/R + L$$

and dividing by L

$$(1/R + w)\,(1 + r) = 1/R + 1$$

$$r/R + w\,(1 + r) = 1$$

Assuming wages paid at the end of the year

$$r/R + w = 1$$

hence

$$w = 1—r/R$$

$$r = R\,(1 - w)^4$$

This shows that, paradoxically, Sraffa found the linear wage–profit relationship, the existence of which he will establish within the Standard System, before even conceiving of a Standard Commodity. The relation here holds for the same reasons as for Ricardo's 'corn', as for any commodity produced only by itself and labour, when we measure wages in terms of this very commodity.[5]

All this reasoning, however, was still based on the highly questionable '**Hypothesis**', and therefore was still under the cloud of the possible objection that it only held under

[1] D3/12/23/1/2.

[2] If $w = 0$, so that $r = r_{max} = R$.

[3] Rather than from assuming that if the rate of profits is zero the rate of wages is equal to 1, Sraffa appears to obtain equation [3] from the equality between the labour embodied in the net product (C_t-C_c) and the direct labour used up in production (L). In fact he writes equation [3] as $C_t - C_c = L$. To its left he adds a question mark, and to its right (probably in reply to that question) he writes: '**(definition of C_t)**'. C_t is defined by Sraffa as the physical quantity produced of the '**general commodity C**'; with an appropriate choice of the unit of measurement (defining the physical unit of a commodity as the quantity which embodies the unit quantity of labour) C_t would also be equal to the labour embodied in this physical quantity C_t. The choice of this unit implies that if $r = 0$, $w = 1$.

[4] D3/12/33/11-12 (dated 5.5.1943). A slight change of notation with respect to that used by Sraffa has been made, to render it uniform with that of *PCMC*.

[5] Provided of course wages are paid *post-factum*.

very special assumptions. A breakthrough seems to happen at the end of January 1944, when we find a remarkable document written from the 27th January to the 1st February (D3/12/36/61–84). Sraffa himself, re-reading it in the 1950s while preparing the final version of the book, marked it as important, and noticed that in it '**The Standard Commodity is first identified**'.[1]

One of the first points Sraffa makes in this document is the following:

The original Hypothesis (that the commods. on right & left are in a constant ratio) and the derived linear relation between r, w, and max r may not be two identical propositions as I have thought so far. The second (which is what really matters to me) may be true when the first is not. The first may be a sufficient but not necessary condition of the second.[2]

Once he realized this, Sraffa needed only a short step to see the conditions required of a commodity for which a linear relationship would hold. In the same document he sees, in fact, three fundamental points:

(1) a composite commodity like commodity C of the above equations—i.e., a commodity produced only by itself and labour—must have a regular reduction to labour (at every stage the quantity of labour employed must be equal to $1/(1 + R)$ times the quantity employed in the following stage);
(2) it is possible, by changing the proportions in which each single commodity is produced (a change which of course does not affect the solutions of the system), to bring the product to a composition which has a reduction to labour which shows that regularity;
(3) as Sraffa had already noticed for Ricardo's 'corn', not more than one commodity can be in that position; if there were more than one when wages are zero the rate of profits could not be uniform: let us imagine that 15 units of corn are produced by means of say 10 units of corn and nothing else (beside labour), and that 20 units of iron are produced with 10 units of iron and nothing else (beside labour). If wages are zero, the rate of profits in the production of corn would be 50%, in the production of iron 100%. There is no room left for prices to bring about a uniform rate of profits.[3]

As Sraffa writes in this very document:

The Standard System is one [we may say] in which *all* commds. occur in the same proportions on the l[eft]**. h**[and]**. s[ide]. and r**[ight]**. h**[and]**. s**[ide]. . . . **In such a system at w=0 there can be only *one* value for r; How can we apply to such a system *another* value of R . . . ? No tampering with prices can possibly satisfy this – not even negative prices.**

And a few weeks later he adds:

[1] On the folder where this document is contained.
[2] The whole of this paragraph was later marked by Sraffa as important.
[3] This is the reason why the example formulated by Besicovitch, which will be discussed below, where it is assumed that there are only two commodities, both produced by only itself and labour, and that wages (paid *ante-factum*) consist of both commodities, is only consistent for $w > 0$.

on the Standard System it is *visible* that only *one* value of R is possible, provided all prices are positive; this is independent of what prices may be, for each commodity occurs in the same proportions on the right & on the left – therefore even if each of the ps could have a number of different values, provided these were all positive, their changing could make no difference to the value of R, which would be unique. Since the Standard System is equivalent to the Actual System, or to any system into which it may be transformed [? By positive multipliers] the value of R must be unique also for all such systems.[1]

These results, however, also raised new questions. A general problem was, after having *seen* these points, to *demonstrate* them, which for Sraffa meant having Besicovitch's approval. Another problem was that of *finding R*, i.e., finding a way to determine its level. In February 1944 (just after the identification of the Standard Commodity) Sraffa formulates a method to determine *R* by means of successive approximations (the same as we find in § 37 of *PCMC*); Besicovitch reformulates it in September of that year.[2]

Interestingly enough, at this point a disagreement between Sraffa and Besicovitch arises, not on the existence, but on the uniqueness of solutions—a question that to a great extent coincided with that of the uniqueness of *R*.

Going a step backwards, it must be mentioned that in September 1943 Besicovitch had formulated an interesting example, where it is assumed that two commodities are produced, each by means of only itself and labour, wages paid *ante-factum*, and a surplus composed of both commodities. Besicovitch showed that giving wages in terms of the net product, every level of *w* (bigger than zero and lower than that for which wages absorb the whole of the net product) is consistent with *two* levels of the (uniform) rate of profits, both positive. The assumptions of Besicovitch's example were certainly peculiar (Besicovitch, however, extended it by continuity to the case where the quantity of each commodity employed in the production of the other is contiguous to 0), but of course it was fully capable, used as a counterexample, to show that uniqueness could not be proved in general.[3] Besicovitch's example could appear as a stumbling block for Sraffa's idea that there was only one solution, but this time Sraffa was not stopped.

At the beginning, Sraffa appears to think he can dispose of Besicovitch's example arguing that in it the '**hypothesis**' did not hold, and that Besicovitch himself allowed that that if the '**hypothesis**' held the solution was unique. A few months later, when in the remarkable document quoted above Sraffa sees the uniqueness of the Standard Commodity, he does not go back to Besicovitch's example in order to reconcile the apparent contradiction between it and the uniqueness he had so clearly seen.

As a matter of fact already at the beginning of February 1944 Sraffa had formulated most of the proof we find in *PCMC* (§41) according to which there can only be one Standard Commodity with all positive components (i.e., only one *R*).[4] On the other

[1] D3/12/36/13, dated 20 February 1944.
[2] D3/12/39/42/3, dated 21.9.1944.
[3] We now know of course that the multiplicity is only apparent: to only one of these values of *r* there correspond non-negative prices.
[4] D3/12/36/vi-vii, dated 13 February 1944; a complete version in Besicovitch's hand was written in May (D3/12/39/3, dated 9 May 1944).

hand, Besicovitch appeared justified by his example in thinking the solution could not in general be unique.

We may notice that Besicovitch's example, which shows that there are multiple solutions for r, was of course consistent with his argument about the 'compound commodities', and in fact Besicovitch (wrongly) thought that the two commodities in his example, each produced only by itself and labour, could also be those '**compound commodities**', both composed of positive quantities of each single component commodity.[1] In fact Besicovitch still in February 1944 maintained (as Sraffa writes) that '**there may be k roots of R, and several may be real, and even (E. & O.E.) several may involve only positive quantities in the corresponding Standard System**'.[2]

The dispute between Sraffa and Besicovitch on the uniqueness of solutions went on for almost a year: until October 1944. A certain lack of communication between them could be justified: Sraffa had a strong point for $w = 0$ (i.e., for $r = R$), which Besicovitch's example, valid only for $w > 0$, could not deny; Besicovitch on the other hand had the strength of an example against which Sraffa had apparently nothing to say—as already mentioned, the point about R did not apply to Besicovitch's example, which assumed $w > 0$, i.e., $r < R$.

In any case, Besicovitch in May 1944 allows that there is only one R, and therefore only one Standard Commodity (with all positive components), and completes the partial demonstration Sraffa had given in February. It is not clear on which basis he was convinced.

The story, however, does not end here, because even when he allows that there can only be one Standard Commodity, i.e., only one composite commodity produced only by itself and labour, and therefore only one solution for the rate of profits and the prices if the wage rate is given in terms of this commodity (because of the linearity of the w-r relationship when w is measured in Standard Commodity), Besicovitch maintains that if wages are measured in terms of any other commodity there are k sets of solutions (if k is the number of the individual commodities produced). Besicovitch's point, which Sraffa at first appears to accept, basically was that giving the wage rate in terms of say a commodity A (different from the Standard Commodity) in a quantity A_λ, given that p_A (the price of A in terms of the Standard Commodity) is not a linear function of r, there could be multiple intersections between the function $A_\lambda\, p_A(r)$, (the value of the given real wage in terms of the Standard Commodity) and the function $w(r)$ (wage in terms of the Standard Commodity). In each of these intersections the given real wage would be equal to a different wage in terms of the Standard Commodity, and it would therefore be associated with a different level of r[3]: there would be multiple solutions for the equation $A_\lambda\, p_A(r) = w(r)$. But Sraffa realizes that Besicovitch's argument does not stand: in a document dated 31 October 1944 and headed '**FINALMENTE**' (AT LAST) in red crayon, Sraffa draws the following figure and notices

[1] Of course we now know that there could only be one composite commodity produced only by itself and labour which has all positive components.

[2] D3/12/36/42, dated 19.2.1944.

[3] D3/12/40/17, dated 14 October 1944.

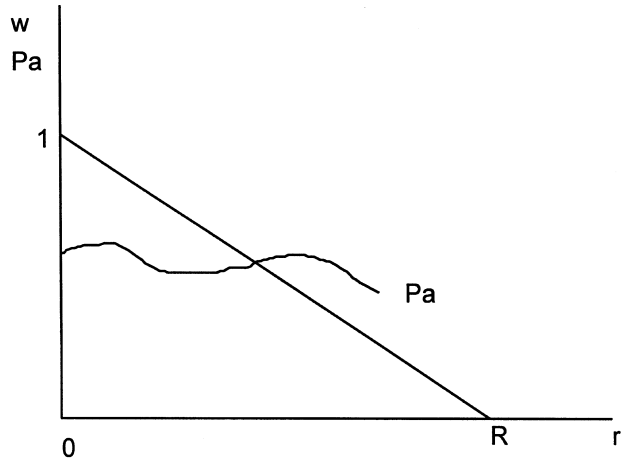

FIGURE 1.

that (on the basis of the reduction to labour of commodity A) its price can be written as

$$\mathbf{p_A = L_0\, w + L_1\, w\, (1 + r) + L_2\, w\, (1 + r)^2 \ldots}$$

adding:

Oggi, dopo dieci mesi di resistenza[1], Besicovich [sic] **ha finalmente ceduto, ammettendo che il sistema ha una soluzione unica nel caso che ogni processo dà un solo prodotto. L'argomento decisivo è stato che la curva del prezzo della merce A non può tagliare più di una volta la retta del salario; perché il prezzo non può mai cadere (in conseguenza di una caduta del salario) in proporzione maggiore del salario; e ciò perché il prezzo può essere espresso in termini di una serie (v. sopra). Il caso in questione è quello in cui una Merce Standard è presa per unità dei prezzi e una merce arbitraria per unità del salario.**[2]

The following day he adds on the verso of the same sheet

Bes[icovitch]**. ammette che la conclusione rimane valida se, invece della Standard Commodity, si prende come unità di prezzi la merce A: poiché si tratta solo di un cambiamento di unità**.[3]

[1] The fact that Sraffa regards January 1944 as the starting point of his discussion with Besicovitch on uniqueness seems to confirm the importance attributed in this paper to the developments of that month, when Sraffa writes the document D3/12/36/61-84, which I have referred to as a remarkable document.

[2] D3/12/40/28 ('Today, after 10 months' resistance, Besicovich [sic] has at last given up, conceding that the system has a unique solution when every process gives a single product. The decisive argument has been that the function of the price of A cannot intersect the wage line more than once; because the price can never fall (in consequence of a fall in the wage) in a higher proportion than the wage itself; this because the price can be expressed as a series (see above). The case in point is when the Standard Commodity is taken as the unit of prices and an arbitrary commodity as the unit of wages').

[3] 'Bes. concedes that the conclusion is still valid if, instead of the Standard Commodity, we take A as the unit of prices: because it is just a question of a change of unit'.

[4] It is interesting that the argument of the gradient had been used by Sraffa a few months earlier in a different context, but he had (temporarily) abandoned it jotting down 'very doubtful' on its margin (see D3/12/36/26/8b, of 16 February 1944).

Sraffa will write again this argument of the '**gradient**'[4] the following day:

w and r vary always in the opposite direction. P_a being capable of being expressed in the form

$$P_a = L_1 w (1 + r) + L_2 w (1 + r)^2 + \ldots$$

where L_s are all non-negative, the gradient of P_a is always less than the gradient of w. And therefore the equation

$$A_L P_a = L w$$

cannot have more than one real root for r.[1]

This, albeit less clear, is the same argument with which part I of *PCMC* ends—and we can also stop here. Besicovitch will for a few months continue giving his 'invaluable mathematical help',[2] but in March 1945 he sends Sraffa the following note:

Dear Sraffa, you asked me recently to help you during the vacation in dealing with your problems. I am very sorry, but I shall definitely be unable to do that. I know that I shall be unable to do anything useful with your problems. In fact, it would be impossible for me to make myself think on these problems. I am very sorry indeed, but I think that under circumstances the best thing is to tell you all that Yours ASB.[3]

I have been unable to find convincing reasons for this rather abrupt end to their relationship. The collaboration, however, was resumed years later, in the final stage of the book's preparation.

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[1] D3/12/63/6, dated 2.11.1944. Sraffa writes down: '**dictated by Besicovitch**'.

[2] As already mentioned, this is how Sraffa will describe it in the preface to his book. In a draft of the preface, which got to proof stage, Sraffa thanked Besicovitch not only for the '**invaluable mathematical help**', but also for 'encouragement and forbearance' (D3/12/112).

[3] D3/12/40/135, dated 17.3.1945.

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