Listen to Sraffa's silences: a new interpretation of Sraffa's Production of Commodities

Ajit Sinha*

This paper argues that the received interpretation of Sraffa's prices as classical 'natural prices' that would prevail when the system is at the centre of gravitation cannot be retained. It offers an alternative interpretation in which Sraffa's prices are completely independent of demand considerations or the condition of equilibrium of demand and supply. The requirement of the uniform rate of profits in Sraffa's price equations is argued to be a logical consequence of the *assumption* that wages are uniform and fixed from outside the system. The paper provides evidence from Sraffa's published and unpublished writings to buttress its new interpretation. It also provides answers to the major criticisms and questions raised against the new interpretation.

Key words: Sraffa, Classical economics, Centre of gravitation, Equilibrium, Price theory, Income distribution, Standard system, Standard commodity *JEL classifications:* B20, B24, B31, B40, B51

1. The problem

In Sraffa's equations, the industrial rates of profits are always taken to be equal. As a matter of fact, Sraffa claims that it *must* be uniform:

[T]he surplus (or profit) *must* be distributed in proportion to the means of production (or capital) advanced in each industry; and such a proportion between two aggregates of heterogeneous goods (in other words, the rate of profits) cannot be determined before we know the prices of the goods ... Accordingly we add the rate of profits (which *must* be uniform for all the industries) as an unknown. (Sraffa, 1960, p. 6, emphasis added)

Address for correspondence: 8 rue Neuve des Boulets, 75011 Paris, France; email: sinha_a99@yahoo.com

* Recipient of a Research Grant from INET and CIGI. I would like to thank the late Professor Pierangelo Garegnani, the then literary executor of Sraffa's unpublished papers, for allowing me to quote from Sraffa's unpublished notes. My thanks are also due to the very friendly staff of the Wren Library, Trinity College, University of Cambridge, where the Sraffa Papers are housed. I would also like to thank all the participants and discussants of my papers at the 2010 conferences on the occasion of the 50th anniversary of Sraffa's book at Meiji University, Tokyo, University of Roma Tre and the University of Bergamo as well as the two anonymous referees for this journal. Finally, a special thanks to Sanjay Reddy for several stimulating discussions on some aspects of the paper.

© The Author 2012. Published by Oxford University Press on behalf of the Cambridge Political Economy Society. All rights reserved.

Manuscript received 14 January 2011; final version received 5 January 2012.

The received interpretation of this condition is that Sraffa *implicitly* assumes his system to be at the classical centre of gravitation; hence, all the rates of profits are assumed to be equal. I find this interpretation unconvincing. There are several prima facie reasons to dispute this:

(i) Sraffa clearly states:

Such a relation is of interest only if it can be shown that its application is not limited to the imaginary Standard system but is capable of being extended to the *actual economic system of observation*. (Sraffa, 1960, p. 22, emphasis added)

Thus it is clear that he deals with an observed actual system and not an *ideal* system that is supposed to exist at the classical centre of gravitation or equilibrium.

(ii) The classical notion of centre of gravitation equilibrates supplies with effectual demands and the process implicitly assumes constant returns to scale, since in the absence of constant returns the supply adjustments towards effectual demands would cause the centre of gravitation itself to move.¹ Sraffa, however, in the very first sentences of the 'Preface' warns the reader not to think in those terms when it comes to his propositions:

Anyone accustomed to think in terms of the equilibrium of demand and supply may be inclined, on reading these pages, to suppose that the argument rests on a tacit assumption of constant returns in all industries ... In fact, however, no such assumption is made. (Sraffa, 1960, p. v)

In the Sraffian literature, the phrase 'equilibrium of supply and demand' is usually interpreted as a reference to neoclassical demand and supply *theory* of prices, and we are encouraged to interpret this supposedly 'awkward' phrase in this manner. It is, however, not clear on what grounds, particularly for such a careful author as Sraffa, the word 'equilibrium' should be translated as 'theory'? But if we refuse to make such a translation, then a serious problem arises for the received interpretation. The leader of the received interpretation, Pierangelo Garegnani, has consistently maintained (and in my opinion correctly) that the classical concept of centre of gravitation or equilibrium survived more or less intact after the marginalist revolution of the 1870s and that it started to go through some changes only after the 1930s:

The study of the permanent effects of changes by means of comparisons between positions of the economic system characterized by a uniform rate of profits was in fact the method used by Ricardo and the English classical economists, when they explained profits in terms of the surplus product left after paying wages at the rate determined by independent economic or social circumstances. But fundamentally the same method was preserved after Ricardo, across the deep change which the theory underwent in favour of a symmetric explanation of profits and wages in terms of the equilibrium between the forces of demand and supply for labour and capital ... It was only in the last few decades that this method, which was centred on 'long-period positions' of the system ... was increasingly challenged ... this departure from tradition has not been due to weaknesses of the method as such, but rather to weaknesses of the dominant theory

¹ Recently, Garegnani has also reluctantly accepted that the classical authors did implicitly assume constant returns in the context of allocation of resources: 'However, Ricardo treated decreasing returns from land, just as Smith had treated the increasing returns from division of labour: as relevant, that is, only for the comparatively large output changes involved in capital accumulation and growth. Unlike what happens in neoclassical theory, Smith and Ricardo *could therefore leave physical returns to scale quite naturally aside when dealing with relative prices in a given position of the economy, with the kind of comparatively small output changes generally involved in that specific analysis' (Garegnani, 2007, p. 188, emphasis added).*

of distribution and, in particular, of the conception of capital it relies on. (Garegnani, 1976, pp. 25-6)

Thus, Sraffa's warning against thinking in terms of 'equilibrium of supply and demand' must equally apply to both classical and neoclassical notions of equilibrium.

(iii) In Appendix B of the book, Sraffa discusses a case of a non-basic good, 'beans', which uses a very large proportion of itself in its production, implying that its rate of profit cannot exceed the ratio of its own net output to its input. Sraffa discusses the problem with the assumption of positive prices for all goods in this case when the rate of profits of the basic goods industries is higher than the one the beans can admit. This problem, however, cannot arise if the system was assumed to be at the centre of gravitation, as the gravitation mechanism would ensure that the bean industry disappears in the process.

(iv) In a note after the publication of *Production of Commodities*, Sraffa wrote:

The wage and the aggregate profit of reality are, at best, rough approximations of the standard wage and profit. But the rate of profit of reality is identical to that of the standard. (PSP, D3/12/111/139, the English translation from the original in Italian is quoted in Gehrke, 2007; emphasis added²)

Clearly, 'reality' must refer to the existing real economy rather than some ideal or average situation prevailing at the centre of gravitation.

As a matter of fact, during the period of his early theoretical breakthrough, i.e. from late 1927 to 1931 (see Garegnani, 2005), we find that Sraffa was worried about how to 'justify or explain the equal percentage added to initial stock of each industry'. And after arguing that capital might not be reinvested in an industry having a lower rate of profit and thus not being able to reproduce itself in the long term, he goes on to add: 'in this way we are allowing to come back through the window the Inotion of cost as] 'inducement' we had excluded from the door [D3/12/6]' (quoted in Garegnani, 2005, p. 475, emphasis added). During the same period, we find Sraffa writing in another note: 'I must find a "force" capable of obliging those people in the market to actuate my equations' (D3/12/7/107-14).³ It should be noted that Sraffa had taken a philosophical or methodological position that the theoretical understanding must be built on only things that are ideally *observable* and, thus, no subjective element (such as 'inducement') should enter his equations. As a matter of fact, there are several notes of this period that show Sraffa's initial struggle with the requirement of the equal rate of profits for the solution of his equations and the notion of constant returns to scale. For example, in a note from 1928, Sraffa writes:

Now I am not assuming any forces: I simply say that, if the values will in reality be as given by the equations certain conditions will be satisfied if not they will not be satisfied. In this case, profits will bear different proportions to capital in different industries. Since this happens to a considerable extent in reality, this means that the values in the market will be different from those in the equations ... I am afraid it will be difficult to make it clear that we are considering

² The original in Italian was earlier cited in Bellofiore and Potier (1998). PSP refers to Piero Sraffa Papers, these are Sraffa's unpublished notes housed at Wren Library, Trinity College, Cambridge. From now on references to these notes will be only referred to by their file numbers, e.g. D3/12/111/139.

³ I am obliged to Nerio Naldi for the English translation of the original in Italian, 'devo trovare una <<forza>> che costringa quella brava gente sul mercato a realizzare le mie equazioni'.

what has actually happened in the markets, and not what might have happened had things been different. It will therefore be useful to explain that the reader may assume that constant returns prevail. (D3/12/7)

The *absence* of the assumption of constant returns to scale, however, was crucial to Sraffa's project, as an introduction of this assumption could leave his system vulnerable to the interpretation that it is a special case of the general equilibrium theory. That is why in the 'Preface' to the book, Sraffa went on to add his recollection that: 'when in 1928 Lord Keynes read a draft of the opening propositions of this paper, he recommended that, if constant returns were *not* to be assumed, an emphatic warning to that effect should be given' (p. vi). Coming back to the same period of his early theoretical breakthrough, we find Sraffa, in an attempt to explain the meaning of his equations, writing:

The significance of the equations is simply this: that if a man fell from the moon on the earth, and noted the amount of things consumed in each factory ... during a year he could deduce at which values the commodities must be sold, *if* the rate of interest must be uniform and the process of production repeated. In short, the equations show that the conditions of exchange are entirely determined by the conditions of production. (D3/12/7, emphasis added)

The reader should take note of the qualifier, '*if* the rate of interest must be uniform'. Interestingly, the qualifier 'if' disappears from the relevant passage in the book! So the question is: what could have happened between the early breakthrough and the publication of the book in 1960? Clearly at this stage Sraffa is conscious of the fact that he needs to 'justify' uniformity of the rate of profits in his system of equations and that the classical centre of gravitation is not an obvious explanation that he can use (for a more detailed criticism of the received interpretation, see Sinha and Dupertuis, 2009; Sinha, 2010A). The answer is: the discovery of the standard system and the standard commodity in the 1940s.

Given that there is a strong prima facie case for rejecting the received interpretation, in Section 2, I provide an alternative argument to justify Sraffa's claim that the rate of profits must be uniform irrespective of the condition of the classical centre of gravitation. In Section 3, I present some evidence from Sraffa's writings (both published and unpublished) showing that Sraffa reasoned in a similar manner. In Section 4, I take up the evidence provided in support of the received interpretation to show that they do not stand up to critical scrutiny. In Section 5, I answer some of the objections raised against my interpretation.

2. Why the rate of profits must be uniform

Let us take an empirical system of production that has produced surplus:

90 tons iron + 120 tons coal + 60 quarters wheat + 3/16 labour \rightarrow 180 tons iron 50 tons iron + 125 tons coal + 150 quarters wheat + 5/16 labour \rightarrow 450 tons coal 40 tons iron + 40 tons coal + 200 quarters wheat + 8/16 labour \rightarrow 480 quarters wheat

¹⁸⁰ tons iron + 285 tons coal + 410 quarters wheat + 1 labour \rightarrow 180 tons iron + 450 tons coal + 480 tons wheat

And in terms of its price equations the system is represented by:

$$\frac{\left(90P_{i}+120P_{c}+60P_{w}\right)\left(1+r_{i}\right)+3/16\,\omega=180P_{i}}{\left(50P_{i}+125P_{c}+150P_{w}\right)\left(1+r_{c}\right)+5/16\,\omega=450P_{c}} \frac{\left(40P_{i}+40P_{c}+200P_{w}\right)\left(1+r_{w}\right)+8/16\,\omega=480P_{w}}{\left(180P_{i}+285P_{c}+410P_{w}\right)\left(1+R\right)+\omega=180P_{i}+450P_{c}+480P_{w}}$$
(1')

Where P and r represent prices and the rates of profits of their respective industries, and ω represents wage rate.

In this system, prices cannot be determined unless the rule for distribution of the surplus is known. Sraffa asserts that the industrial rates of profits *must* be uniform. If that is so, then given wages, the two relative prices and the uniform rate of profits of the system could be simultaneously determined. It has been almost universally interpreted that Sraffa's claim that the rate of profits must be uniform is an admittance of the competitive equilibrium condition or the condition of the centre of gravitation (see John Hicks for an exception).⁴ Without going into exegetical arguments that Sraffa did not think in terms of equilibrium of demand and supply, let me here motivate a logical argument behind the condition of the uniformity of the rate of profits independently of the notion of equilibrium of demand and supply. Below I show that if wages are taken to be fixed from outside and are taken to be uniform (or the heterogeneous labours are homogenised by the given wage differentials, as in Sraffa's examples), then a logical corollary of it is that prices must be such that all industrial rates of profits *must* be equal in any system of basic goods,⁵ as long as prices are determined by the system of equations and not taken to be fixed from outside.

The proof of the above proposition is simple. Let us assume that wages are fixed at zero, then in the equation system (1') we have three independent equations and five unknowns—two relative prices and three industrial rates of profits, given $\omega = 0$. Unless two industrial rates of profits are given, we cannot determine the two relative prices from within the equation system. But the rates of profits cannot be given independently of prices as they are dependent on prices. Can we, however, determine the average rate of profit of the global system (i.e. *R*) from the given information, instead of determining the individual industrial rates of profits? The answer is yes. Because, whatever turns out to be the average rate of profit, the mathematical property of the

⁵ A basic good is a good that inters directly or indirectly as input in the production of all the commodities, whereas a non-basic good does not inter directly or indirectly in the production of any basic good, though it could inter as input in the production of the subset of non-basic goods.

⁴ 'Sraffa leaves us to find out what his prices are, but I doubt if they are equilibrium prices. They seem to be prices which are set upon products, by their producers, according to some rule. Now it is perfectly true that we are nowadays familiar with that method of price-fixing, by "mark-up"; but when that method is used, the rate of profit that is used to establish the mark-up is conventional. Now it may be that Sraffa wants us to think of his rate of profit as being conventional; and that the uniformity of the rate of profit throughout his system, of which he makes so much, is just a uniformity of convention' (Hicks, 1985, p. 306). Among the Sraffians, Roncaglia (1978, p. 16) did appreciate that 'there is no reason to believe that Sraffa's prices of production should equate quantity demanded and quantity supplied'; however, he could not come up with an argument that would justify the equality of the rate of profits in Sraffa's system and thus succumbed to holding the contradictory position that Sraffa's system was 'a photograph of the market place' (an expression Sraffa uses in his unpublished notes of the period 1927–31) as well as that his system was assumed to be at the centre of gravitation (also see Roncaglia, 2000).

(2)

1328 A. Sinha

average ensures that it can be equally distributed over the total capital. Therefore, if we *assume* that all the rates of profits were equal, then it reduces the number of unknowns to three and we could determine this rate of profit, which must be equal to the average rate of profit *R*. The point to note is that we do not know whether this average rate of profit is equally or unequally distributed in the system and, therefore, we still do not know whether the prices associated with an equal rate of profits hold in the given real system or not. To solve for the industrial rates of profits and prices, we need to convert our system of equations to its standard counterpart.

Let us assume an imaginary system given by:

120 tons iron + 160 tons coal + 80 quarters wheat + $\frac{1}{4}$ labour \rightarrow 240 tons iron 40 tons iron + 100 tons coal + 120 quarters wheat + $\frac{1}{4}$ labour \rightarrow 360 tons coal 40 tons iron + 40 tons coal + 200 quarters wheat + $\frac{2}{4}$ labour \rightarrow 480 quarters wheat 200 tons iron + 300 tons coal + 400 quarters wheat + 1 labour \rightarrow 240 tons iron + 360 tons coal + 480 tons wheat

And in terms of its price equations, the system is represented by:

$$\frac{\left(120P_{i}+160P_{c}+80P_{w}\right)\left(1+r_{i}\right)+\frac{1}{4}\omega=240P_{i}}{\left(40P_{i}+100P_{c}+120P_{w}\right)\left(1+r_{c}\right)+\frac{1}{4}\omega=360P_{c}} \frac{\left(40P_{i}+40P_{c}+200P_{w}\right)\left(1+r_{w}\right)+\frac{2}{4}\omega=480P_{w}}{\left(200P_{i}+300P_{c}+400P_{w}\right)\left(1+R^{*}\right)+\omega=240P_{i}+360P_{c}+480P_{w}}$$
(2')

Production system (2) is nothing but Sraffa's standard system to the given empirical system (1). It redistributes the total labour of the system or rescales the real system in such a way that the aggregates of its inputs and outputs come out in the same proportions. Let us again assume that wages are zero, then in the above given example of equation system (2'), it is clear that the rate of profit of the system as a whole, i.e. R^{\star} is equal to one-fifth or 20%. This is because in this case the ratio of the aggregate physical net output to the physical aggregate inputs can be known without the knowledge of prices, since it is a ratio of heterogeneous goods made up in the same proportion. This ratio is completely independent of prices-no matter what prices prevail, it will not affect the global rate of profit (i.e. R^{\star}) of the standard system. It is, however, a mathematical property of the standard system that R^{\star} is always equal to the average rate of profit (R) of the real system, as derived above. Since the individual equations of systems (1') and (2') are the same, with only difference in their weights in the total, it is clear that R will always be equal to R^* (i.e. R of all the possible rescaled systems of the given standard system will always be equal to R^*), if and only if all the r values in equation system (1') must be equal.

Now we drop the assumption that wages are zero. Let us define a money commodity as a composite commodity made up of the three basic goods in the same standard proportion, say we define: $(40P_i + 60P_c + 80P_w) = 1$. This is our standard commodity. If now we give wages in the standard commodity, then for every given wages from 0 to 1 we can discover the average rate of profit of the standard system (R^*) associated with those wages independently of prices, as R^* is directly determined by the physical ratios.

The relationship between the wages in terms of the standard commodity (ω) and R^{\star} is given by: $R^* = R^*_{max}(1 - \omega)$. This relationship is a structural property of the system of equations. It proves that the distribution of income between the two classes in terms of wages and the rate of profits on total capital is independent of prices. The point to note is that the system of equations does not admit any other distribution of income. As we have shown above, since the empirical system (1) is nothing but an equivalent system of the standard system (2), as their size and equations are the same but only arranged in different proportions, this relationship between R and ω must apply to the empirical system (1) as well. This implies that as long as wages are given in terms of the standard commodity, R^* must be equal to R and therefore all the industrial rates of profits in the empirical system must be equal for all given values of ω . It is interesting to note that prior to the publication of the book, Sraffa had written a couple of slogans that captured the spirit of the book, but he finally decided not to use them. One of the slogans was: 'A Dividend could be declared before knowing what is the price of the company's product.' The other slogan was: 'The St. Syst [Standard System] provides tangible evidence of the rate of profits as a non-price phenomenon' (PSP, H2/89, f. 56; quoted in Pasinetti, 2001).

The structural property of the system, as revealed above, can also be illustrated in another manner. Take the empirical system (1'). Its net output-capital ratio is given by (165 tons coal + 70 quarters wheat)/(180 tons iron + 285 tons coal + 410 quarters wheat). Though this ratio is not well defined without knowledge of the prices, it is clear that it is a technical relation of the system and any change in the distribution of the net output between the workers and the capitalists should not affect the value of this technical ratio. Now, on one hand, it is clear that if the distribution of net income affects the relative prices, then in most circumstances it will affect the value of the net output-capital ratio, since the physical composition of the net output is not the same as the physical composition of the capital. On the other hand, it is also clear that if prices were not affected by changes in the distribution of income, then the value of the net output-capital ratio would also remain unaffected. Below, we first argue that relative prices cannot remain constant against changes in the distribution of income (of course, in the systems with unequal ratios of industrial means of production to labour, such as our system (1)). Therefore, the constancy of the net output-capital ratio cannot be maintained on the basis of constancy of prices. We then argue that for the net outputcapital ratio to remain constant, the changes in prices must be such that $R = R^*$ and therefore the industrial rates of profits are always equal.

Let us take system (1) and begin with wages equal to the net output (i.e. $165P_c + 70P_w$) and, therefore, R = 0. In this case it is a technical requirement of the system that all the *r* values are also equal (i.e. = 0). This is because if any *r* were to be positive, some *r* have to be negative, which would imply that the whole system was economically unviable. In this case the solution to the set of prices exists; as is well known, the prices will be in the ratios of their labour values. Let us put $(165P_c + 70P_w) = 1$. Now rescale the system to its standard proportion. We know that the solution of a system of equations does not change by rescaling the system. Thus, the same labour values or *P* and *r* values will follow for the standard system (2'). From this it follows that $(40P_i + 60P_c + 80P_w) = (165P_c + 70P_w) = 1$, when $R = R^* = 0$. Now let us reduce wages by half and assume that this has no impact on relative prices. These prices would give rise to unequal rates of industrial profits in both the systems, as the ratios of means of production to labour in all the industries are not uniform. These prices would also generate a

value for R, which in our example turns out to be about 10.5%. Now, reduce wages by half in the standard system (2') as well. Since prices have remained constant, wages in the standard system are given by $\frac{1}{2}(40P_i + 60P_c + 80P_w)$. This wage is associated with a value for $R^{\star} = 10\%$. This rate, however, is not contingent on the labour-value prices. No matter what prices prevail, if the wage is given by $\frac{1}{2}(40P_i + 60P_c + 80P_w)$, then the value of R^* must be 10%. Among all possible prices, there must be at least one set of prices that would be a solution for the real system for the wages given by $\frac{1}{2}(40P_i +$ $60P_c + 80P_w$, if the real system has a solution. Thus, if wages in system (1') are taken to be equal to $\frac{1}{2}(40P_i + 60P_c + 80P_w)$, then its price solution must generate R = 10%, as shown above. However, as we have calculated above, if prices remain at their labour values, then wages given by $\frac{1}{2}(40P_i + 60P_c + 80P_w)$ generate the value of R equal to about 10.5%—remember, since prices have remained constant at labour-value ratios, $\frac{1}{2}(40P_i + 60P_c + 80P_w) = \frac{1}{2}(165P_c + 70P_w)$ —which contradicts the mathematical solution of the system. This proves that in a system where the ratios of industrial means of production to labour are unequal, relative prices cannot remain constant when the distribution of income changes.

Next, I show that the ratio of net output to capital remains constant if R is always equal to R^* . Let us assume that wages are paid or measured in standard net product in both the standard system as well as the real or empirical system. Let us also normalise the standard net product to one, i.e. $(40P_i + 60P_c + 80P_w) = 1$. Let us give wages in the standard system from 1 to 0 and plot the resulting R^* values. We will plot a straight line relationship between ω and R^* , with $R^*_{max} = 20\%$ when ω is zero. If R in the real system is always equal to R^* , then it is clear that we would draw exactly the same relationship between R and ω in the real system as well. The general form of this straight line relationship is given by $R = R_{max}(1 - \omega)$. This implies that R_{max} is equal to $R/(1 - \omega)$, which is a constant as it is a slope of a straight line. But R_{max} is nothing but the ratio of the value of net output to the value of aggregate capital, i.e. the net output–capital ratio. Hence we have shown that the condition of equality of global rates of profit ' R^* of the empirical system is the technical requirement of the empirical system, and this technical requirement can be fulfilled if and only if all its industrial rates of profits are equal.

This is a remarkable result. It shows that the production equations of basic goods along with the knowledge of wages in terms of the standard commodity provide sufficient information to determine the prices irrespective of the demand conditions. Here I should point out that Sraffa's propositions are not built on the usual or mechanical cause-and-effect relationships. All the dependence and changes in variables in Sraffa's propositions describe logically necessary relationships between those variables, such as a change of 10° of an angle in a Euclidean triangle must be associated with 10° combined changes in the angles of the other two angles in the opposite direction.⁶

One interesting implication of our above argument is that the prices and the industrial rates of profits of at least basic goods industries of an economic system are completely independent of the market structure. No matter whether an industry is characterised by perfect or imperfect competition or pure monopoly or oligopoly, the price and its rate of profit are determined by the structure of all the interdependent industries and

⁶ Sen (2003, p. 1253) has also argued that '[t]he temptation to see Sraffa's contribution as a causal theory of price determination ... must be resisted ... The sense of "determination" invoked by Sraffa concerns the mathematical determination of one set of facts from another set'.

their combined productivity or the productivity of the system as a whole. Individual industries have no independent existence. This is why we find that the word *competition* (or any kind of market structure for that matter) simply does not appear in the entire book of Sraffa. It is quite intriguing that a book devoted to the theory of value, and that too by an author who had made significant original contribution to the theories of market structures, should remain absolutely silent about the market structure the theory applies to. Now the reason for this silence appears to be simple: market structures are simply irrelevant to the problem of prices and the rate of profits of a system of basic goods, if the distribution of income is given from outside.

Another interesting aspect of our interpretation is that for the first time the nature and significance of the first chapter of Sraffa's book and the controversy between Harrod (1961) and Sraffa (1962) on this chapter become clear. There has been almost no discussion in the Sraffian literature on the question of why Sraffa begins his book with a short chapter on subsistence economy. The conclusion of Sraffa's analysis of the subsistence system is that 'such values spring directly from the methods of production' (Sraffa 1960, p. 3). The example of subsistence economy that Sraffa begins with is given by:

280 quarters wheat +12 tons iron \rightarrow 400 quarters wheat 120 quarters wheat +8 tons iron \rightarrow 20 tons iron

This system is in a self-replacing state. This led Harrod (1961) to conclude that Sraffa apparently derives the relative prices of commodities in his two-good subsistence model by the ratio of the respective excess productions of the two sectors, which leads him to his major criticism of Sraffa's equations:

In an early passage (p. 7), where he is still dealing with a two-commodity world of wheat and iron, he assumes that the whole net income is taken out in wheat. That may seem sensible, as consumers do not presumably desire iron as such. But there is nothing in this passage to require that the second commodity, iron, is specifically a capital good. On the contrary, it is supposed to be setting the matter out in a perfectly general way. This is a difficulty arising, at the very outset, from the neglect of the composition of consumer demand. If consumers did happen to wish to have some iron, that would at once, in accordance with Mr. Sraffa's own equations, affect the price ratios, which his system purports to be determining without reference to consumer demand. I believe that this objection runs through all the complications of his subsequent treatment. (Harrod, 1961, p. 78)

To which Sraffa (1962) responded:

Now this is clearly a misunderstanding, since the exchange ratios are, of course, determined by the equations of production and not by the ratios between the excess productions of the commodities. Sir Roy has been misled by the fact that the two ratios *happen* to be equal in the first example given (a no-surplus two-commodity system which is in a self-replacing state). Even in this simple case, however, if, with the same equations, the two commodities were produced in different proportions (so that the system ceased to be in self-replacing state) the exchange ratio would remain the same but the ratio between the excess productions of the two commodities would be changed, so that the two would no longer be equal. (Sraffa, 1962, pp. 477–8, emphasis added)

The point to note is that the idea that production equations determine exchange values is not dependent on the condition of the equilibrium of demand and supply (the self-replacing state). Let us suppose that the iron sector is twice as large; thus, the system is no longer in a self-replacing state. There is excess demand for wheat and

excess supply of iron. However, even in this circumstance of disequilibrium of demand and supply, the reader can easily check that the exchange ratio between iron and wheat *must* remain 1 ton of iron for 10 quarters of wheat, since for a subsistence economy the value of the net output of any industry must be equal to the value of all its input requirements. And this condition would hold for the general case of an *n*-good subsistence economy. This result reveals the reason for starting the book with an analysis of the subsistence economy. Sraffa's point happens to be that this fundamental result remains valid for surplus-producing economies as well; the only difference is that in this case the result is not readily evident.

3. Evidence from Sraffa's writings

Below, I produce some evidence from Sraffa's writings that seems to support the argument presented in the above section. In the *Production of Commodities*, Sraffa seems to be arguing in a similar manner when he declares that the mathematical properties of the standard system commute to the real system:

But the actual system consists of the same basic equations as the Standard system, only in different proportions; so that, once the wage is given, the rate of profits is determined for both systems regardless of the proportions of the equations in either of them. Particular proportions, such as the Standard ones, may give transparency to a system and render visible what was hidden, but they cannot alter its mathematical properties. (Sraffa, 1960, p. 23)

The reader should note that the classical condition of supplies equal to the effectual demands cannot be a mathematical property of the system—it is supposedly a behavioural property of the system. It should also be noted that Sraffa could not implicitly assume that supplies were equal to their effectual demands for both the real and the standard systems—it would be bizarre to assume that the effectual demands were in standard proportion even in an imaginary world. Thus, Sraffa could not impose the condition of a uniform rate of profits on his standard system on the basis of the so-called implicit assumption that the system is at its centre of gravitation. Hence the rate of profit of the standard system R^* and the claim is that the two global rates (R^* and R) must always be equal as long as the wages are measured in the standard commodity. It is the proposition regarding the equality of the global rates of profit of the rescaled systems that allows Sraffa to directly deduce that all the industrial rates of profits *must* also be uniform in the two systems, as argued above. This point becomes clearer in the very next paragraph from the above-quoted passage:

The straight-line relation between the wage and the rate of profits will therefore hold in all cases, provided only that the wage is expressed in terms of the Standard product. The same rate of profits, which in the Standard system is obtained as a ratio between *quantities* of commodities, will in the actual system result from the ratio of aggregate *values*. (Sraffa, 1960, p. 23, original emphasis)

The reader should note that both the ratios of 'quantities of commodities' and of 'aggregate values' are well defined only at the *global* level and have no meaning at the local or industrial level.

Further on, in his unpublished notes written in 1955, we find that Sraffa invokes similar reasoning behind the possibility of an existence of a standard commodity:

With changes in w-

The impulse towards price change is an internal one to each industry. It arises from its own internal conditions—not from those conditions *compared* with those of other industries. Hence the possibility of an *invariable* commodity. (D3/12/59, original emphasis)

Recall the discussion on the standard commodity in Sraffa (1960). He starts with zero profits and all income going to wages. Then wages are reduced by a certain percentage. Sraffa's argument is that this gives rise to a positive global rate of profit and all the industrial rates of profits equal to it. Given that all the rates of profits must be equal, the old prices applied to the goods create surpluses and deficits in the industries given their different proportions of labour and means of production. And it is these surpluses and deficits that force the industries to adjust their prices. That is why an industry that will not have any surplus or deficit will have no compulsion to change its price and, hence, there is possibility of an 'invariable commodity'. If one allows the gravitation mechanism to explain the equality of the rate of profits in the system, then no commodity could stay invariant. The difference between the two approaches is this: in Sraffa's case, the condition of equal rate of profits is given or must be applied to the system in all the circumstances and prices change as a consequence of this condition. Hence, the idea of change based on comparison with other industries is categorically denied. In the classical case, on the other hand, the rates of profits eventually become equal as a *consequence* of changes in prices, which are explained precisely in terms of *comparison* with other industries. In this context it is instructive to refer to the quotation from Sraffa's notes of 1928, quoted above on pp. 1325–6 (D3/12/7). Here, Sraffa had tried to interpret his equations and the condition of the equal rate of profits as the classical/orthodox equilibrium condition without the forces that are supposed to bring the system to equilibrium. In this context, he goes on to argue in the usual manner that non-equilibrium prices would result in unequal rates of profits. But then he realises that his equations would be meaningful only if constant returns are assumed. A clear shift in Sraffa's position on the condition of uniform rate of profits in his system of equations is evident here.

Further on, Sraffa in another note of 1955 writes:

the rate of profits at the various individual levels of w will be r = R(1 - w). Individual prices will move in all directions with the variation of w, but here again prices will make no difference: r is a ratio between two quantities of the same composite commodity and can actually be discovered before knowing what those prices are. The rate of profit is embedded 'in the things' and no manipulation of prices could ever affect it. [There could be no more tangible evidence of the rate of profits [being, as] a non-price phenomenon (effect)]. (D3/12/53, all underlines, parentheses and brackets are original)

Yet again it is claimed that the real rate of profits must be identical to the standard global rate of profit. This finding shows that uniformity of the rate of profits in the system has nothing to do with the equalisation of the supplies with their effectual demands.⁷

⁷ Joan Robinson (1961) had come closest to understanding this as she claimed that the 'clue' to understanding *Production of Commodities by Means of Commodities* could be found in the 'corn model' of Sraffa's (1951) 'Introduction' to Ricardo's *Principles*. In the 'corn model', e.g. 1 ton of corn produces 1.5 tons of corn, the rate of profit is 50% no matter what is the final demand for corn. This physical relationship between inputs and outputs that is palpably evident in a single basic-good model is obscured in *n*-basic goods model. But Sraffa's analysis with the help of the standard system reveals that the insight of the corn model remains valid in a more general case as well.

As a matter of fact, relative prices cannot go anywhere they like—they are completely constrained by the system of production and distribution. In some sense Sraffa's result points to a similar break in economics as the break from classical mechanics to quantum mechanics.⁸ The classical and neoclassical economics treat individual industries as independent entities, which through their interaction generate centres of gravitation that bring a system into being. Sraffa's result shows that the system is not made up of independent industries, but must be treated as an interconnected whole unit and the properties of the whole determine the properties of its parts.

4. A critique of the evidence provided by the received interpretation

Now let us look critically at the evidence provided in support of the received interpretation that Sraffa's outputs are at the centres of gravitation. We may be asked: if what we say above is true, then what could Sraffa mean by his statement in the 'Preface' where he states: 'This standpoint [i.e. of given output], which is that of the old classical economists from Adam Smith to Ricardo, has been submerged and forgotten since the advent of the 'marginal' method' (p. v), as the gravitation mechanism was clearly part of Adam Smith's and Ricardo's systems. The answer to this question can be found in one of Sraffa's notes of the period of his early breakthrough:

When A. Smith etc. said '*natural*' he did not in the least mean the 'normal' or the 'average' nor the 'long run' value. He meant that physical, truly natural relations between commodities, that is determined by the equations, and that is not disturbed by the process of securing a greater share in the product. (D3/12/11, quoted in Garegnani, 2005, p. 474, original emphasis)

Clearly, from the beginning of his new theoretical adventure, Sraffa had completely discounted the notion of 'centre of gravitation' as part of the 'classical standpoint'. The reader should note that we are here not concerned with the 'correctness' of Sraffa's reading of Adam Smith. The evidence shows that when Sraffa uses the word 'natural price' of classical economists, he is not using it as the long-term equilibrium or centre of gravitation price. It should also be noted that in his lecture notes of 1928, Sraffa spends a lot of time on the classical theory of value. However, it is the *objective* aspect of the classical theory of value that is emphasised there and the notion of the centre of gravitation is completely ignored.

The second alleged evidence is that Sraffa also refers to the approach of his book being 'reminiscent of certain points of view taken by the old classical economists from Adam Smith to Ricardo'. They are all listed in Appendix D of the book: (i) Quesnay's *Tableau Economique* is credited for the circular point of view; (ii) the notion of basic goods could be discerned in Ricardo's 'corn model'; (iii) the idea of the standard commodity could also be discerned in Ricardo; (iv) the notion of maximum rate of profits is found in Marx; and (v) the treatment of fixed capital as a kind of joint product could be found in Torrens. Interestingly, we find that there is

⁸ It may be noted that Sraffa was well aware of the developments in quantum mechanics. As early as 1928 he had noted down a passage from H.S. Allen's paper on 'The Quantum Theory' published in *Nature*, where Allen writes, 'Heisenberg put forward the demand that only such quantities as are observable should be represented in the mathematical formulation of atomic theory ... This led to the development of the matrix mechanics, every term in a matrix corresponding to something which is, at least ideally, observable.' Of course, Sraffa makes the same demand from economic theory.

no reference to the notion of 'natural prices' or the 'centre of gravitation' in the list. If Sraffa had accepted the notion of centre of gravitation in his book, then the question is: why did he not acknowledge Adam Smith for this idea? Why would he ask the reader not to bring the baggage of the thinking in terms of equilibrium of demand and supply in the very first sentence of the 'Preface', and then go on to implicitly assume it throughout the book?

Anyway, the most important evidence that is invoked in favour of the received interpretation is Sraffa's statement in the book that 'Such classical terms as "necessary price", "natural price" or "price of production" would meet the case, but value and price have been preferred as being shorter and in the present context (which contains no reference to market prices) no more ambiguous' (Sraffa, 1960, p. 9). A close reading of this passage, however, confirms our interpretation and rejects the received interpretation. As we have shown above, quantitatively Sraffa's price is the same as Smith's and Ricardo's 'natural price'; however, Sraffa's price is not defined to hold only at the centre of gravitation. Thus it does not need any reference to 'market prices'. Sraffa's caveat, that his context 'contains no reference to market prices', takes away the essential element of the gravitational mechanism. It is the 'market prices' that gravitate towards the centres of gravitation in the classical system. What meaning could be assigned to a concept whose essential compliment is deliberately left out of the theoretical context? Can we imagine a centre of gravitation in a space without matter? As a matter of fact, Sraffa simply dissolves the classical distinction between the 'market' and the 'natural' prices, just like sugar in water.

5. My response to some objections

It is frequently argued by the Sraffians that Sraffa's prices cannot be the 'market prices', as 'market prices' are influenced by innumerable causes and thus cannot be 'determined'. Therefore, Sraffa's prices must be the classical 'natural' prices. This argument implicitly assumes that the classical distinction between 'market' and 'natural' prices and the theory of gravitation are obviously true; therefore, any theory of price determination must refer to the 'natural' or some sort of 'equilibrium' or 'average' prices. Such objections, however, fail to notice that it is precisely the classical distinction between the 'market' and 'natural' prices and the theory of gravitation that are challenged by our interpretation of Sraffa's theory. Thus they cannot constitute the ground for an objection to our interpretation.

At various conferences I have frequently come across such comments as: 'what prices are they; they cannot be the prices prevailing in the shop downstairs?' Now, as far as the basic goods are concerned, the prices are the well-defined ratios of exchange between the industries. In any case, Sraffa explicitly assumes an annual harvest cycle and a market after the harvest: 'We retain however the supposition of an annual cycle with an annual market' (Sraffa, 1960, p. 10). Thus the market is well defined. So the 'shop downstairs' is nothing but a red herring. The real question is whether Sraffa's prices are supposed to be the actual prices at which industries exchange their products in the annual market or whether they are the prices that would prevail when all the industries' supplies would be equal to their respective effectual demands? The latter position involves reasoning in terms of counterfactuals. Sraffa, however, consistently refrains from counterfactual reasoning. In this context, it is again instructive to recall Sraffa's note quoted on p. 1325–6: 'I am afraid it will be difficult to make it clear that

we are considering what has actually happened in the markets, and not what might have happened had things been different' (D3/12/7).

Another question that I have been asked is: if the uniform rate of profits is a logical consequence of Sraffa's equations, then how do we explain the real differences in the rates of profits that exist in the real world? The answer to this question has several folds. First, it is never very prudent to compare a theoretical category directly with the data available from either government or private sources. It should also be kept in mind that Sraffa's prices must be applied to both the inputs and the outputs simultaneously, whereas company accounts calculate their input costs at the prices existing in the previous period compared with their output prices. This procedure of calculating profits can not only give differences in the profit rates, but can also obscure the physical nature of the rate of profits, which is what Sraffa was trying to reveal. The point becomes clear when we look at the standard system. In the standard system the standard ratio is determined by the physical input-output data. Now, no matter what sets of prices are applied, as long as they are applied to both the inputs and the outputs, this ratio will not be affected. However, if we apply a different set of prices to the inputs and a different set of prices to the outputs, then such a standard ratio would contradict the physical ratio. This shows the unscientific nature of the calculations of the rates of profits by firms. Furthermore, Sraffa's equations are for well-defined *industries*, they do not rule out differences in the rates of profits among firms within an industry. In the end it should, however, be noted that there is no foreign trade in Sraffa's system; to what extent foreign trade in a basic good could affect the rates of profits of various industries needs to be studied.

I have also been asked: if you deny price mechanism then how do you account for market adjustments for excess demands and supplies; in a bad harvest year isn't the rise in prices the result of excess demand for food? My answer is simple. When there is excess demand or supply of a commodity, the inventory of the commodity either falls or rises; however, the inventories are accounted for by the same prices. In the next production cycle, the producers either decide to expand or contract their outputs depending on their inventory management. Such movements in supplies of basic goods can change the whole set of prices in the next production cycle if constant returns do not prevail. But the prices of the next production cycle would again be determined by the new Sraffa equations and not by the demand and supply prevailing in the market. As far as the rise in prices of food in a bad harvest year is concerned, it is a direct case of change in Sraffa's equation for food from a normal year to a bad year and its consequence on prices.

At a conference a discussant argued that I put too much emphasis on the standard system and the standard commodity; did not Sraffa himself acknowledge that 'The Standard system is a purely auxiliary construction' (Sraffa, 1960, p. 31)? As a matter of fact, too much emphasis on the standard system is not mine but Sraffa's own. In a slender classic of 87 pages, a work that appears to be consciously designed as a piece of minimalist art, Sraffa devotes 25 pages of the main text and 13 separate references in the Index to the problems related to the standard system and the standard commodity. Anyone who has seen Sraffa's unpublished notes of the early 1940s cannot remain unimpressed by the time and intensity of energy devoted to this problem by Sraffa. Yet within the context of the received interpretation of Sraffa's book, it is difficult to understand the significance of the standard system for his project (this is also the case with the neoclassicists's interpretation of Sraffa's book). So a story has been concocted that Sraffa's search for the standard system and the standard commodity was an attempt

to solve Ricardo's problem of 'invariable measure of value'.⁹ However, to suggest that an author of Sraffa's heightened sense of aesthetics would devote one-third of his book, and that too in the middle of it, to a side issue or a solution to someone else's problem without any notice to the reader is simply preposterous. But not only that, it flatly contradicts Sraffa himself. In Appendix D, 'References to the Literature', Sraffa takes pain to point out in a parenthetical remark that 'It should perhaps be stated that it was only when the Standard system and the distinction between basics and non-basics had emerged in the course of the present investigation that the above interpretation of Ricardo's theory suggested itself as a natural consequence' (Sraffa, 1960, p. 93, emphasis added). Thus it is clear that the discovery of the standard system was an integral part of the 'present investigation' and a particular interpretation of Ricardo's theory was an afterthought. Finally, I should point out that 'auxiliary' does not mean 'unimportant', as it is suggested in the Sraffian literature; it means '(someone or something) giving help or support, esp. to a more important person or thing', according to the Cambridge International Dictionary. And this is the exact sense in which Sraffa uses the word when he writes that 'The Standard system is a purely auxiliary construction'. Sraffa's sense is that the standard system stands in relation to the real system as a scaffolding stands in relation to a building. The scaffolding is an essential support for the building, without it the building could not be constructed; however, once the building is constructed, it can be taken off. Similarly, the mathematical properties of the real system could not be understood without the support of the standard system, but once the problem is solved, it can be removed from further analysis of the real system.

A referee of this journal commented:

It [i.e. my interpretation] does not try to argue why it would be useful to have prices according to the new interpretation—why anyone should be interested in such a measure. The title of Sraffa's book suggests another orientation. It is called 'prelude to a critique of economic theory'. Hence, the prices must serve as a basis for such a critique and must therefore be related in some meaningful way to the prices used in economic theory, in particular in neoclassical theory. The advantage of the standard interpretation is that it directly addresses long-run prices which can be related to prices as a measure of capital goods in order to criticise neoclassical theory or to prices of production in Marx, in order to see the transformation problem in a new perspective.

First, if a theory of prices that claims to be valid in a particular circumstance (in this case the highly unlikely scenario when supplies happen to be equal to demands) serves a purpose, then how can it be denied that a theory of prices that claims those prices must hold irrespective of that particular circumstance, but by definition including that particular circumstance, must also serve that purpose? A general theory, which shows another theory to be just a particular case of it, is always a more powerful theory. Furthermore, it is simply incorrect to suggest that Sraffa's book was designed to show that the rate of profits or interest cannot be explained by the theory of marginal productivity of capital. If that was the case, then the purpose was achieved by showing the possibility of reswitching techniques. There would be no reason to subtitle the book as 'a *prelude* to a critique of economic theory', as the intended critique was already achieved in the book (for details on the nature of Sraffa's last chapter on 'Switch in Methods of Production', see Sinha 2010A). As a matter of fact, Sraffa clearly notes in the 'Preface' that:

⁹ See Sinha (2010A, 2010B) for an alternative interpretation of Ricardo's problem of the 'invariable measure of value'.

It is, however, a peculiar feature of the set of propositions now published that, although they do not enter into any discussion of the marginal theory of value and distribution, they have nevertheless been designed to serve as the basis for a critique of that theory. *If the foundation holds, the critique may be attempted later, either by the writer or by someone younger and better equipped for the task.* (Sraffa, 1960, p. vi, emphasis added)

How much clearer Sraffa has to be for his followers to understand that the book, as such, is not a critique of economic theory. It is an attempt to establish a foundation of an alternative theory, which might in future, if the foundation holds, serve as a basis for launching a critique of economic theory. Hence it is a 'prelude' and not a 'critique'. This is the point our interpretation establishes. It shows that Sraffa has succeeded (at least in the case of a system of basic goods) in establishing a theory of prices that is completely independent of human psychology or demand considerations, which is the basis of modern economic theory. It is now up to us (i.e. those who are not satisfied by the modern economic theory) to see what kind of critique can be launched from this foundation. In this context, let me begin by pointing out that in Sraffa's equations prices have only one function in the system and that is to consistently account for the given distribution of income after the harvest. Sraffa, however, nowhere states that the distribution of income between the two classes *must* be given from outside. What he says is: 'The result of adding the wage as one of the variables is that the number of these now exceeds the number of equations by one and the system can move with one degree of freedom; and *if* one of the variables is fixed the other will be fixed too' (Sraffa, 1960, p. 11, emphasis added). The reader should note the qualifier 'if' in the above quotation. It is now for us to develop grounds for a defence of this proposition. It is my sense that the next battle between the modern general equilibrium theory of prices and the Sraffian theory must take place on the qualifier 'if'.

Bibliography

Allen, H.S. 1928. The Quantum Theory, Supplement to Nature, no. 3084, pp. 887-95

- Bellofiore, R. and Potier, J.-P. 1998. 'Piero Sraffa: nuovi elementi sulla biografia e sulla ricensione di *Produzione di merci in Italia', Il pensiero economico italiano*, vol. 6, no. 1, 55–103
- Garegnani, P. 1976. On a change in the notion of equilibrium in recent work on value and distribution: a comment on Samuelson, pp. 25–45 in Brown, M., Sato, K. and Zarembka, P. (eds), *Essays in Modern Capital Theory*, Amsterdam, North Holland
- Garegnani, P. 2005. On a turning point in Sraffa's theoretical and interpretative position in the late 1920s, *European Journal of the History of Economic Thought*, vol. 12, no. 3, 453–92
- Garegnani, P. 2007. Professor Samuelson on Sraffa and the classical economics, *European Journal of the History of Economic Thought*, vol. 14, no. 2, 181–242
- Gehrke, C. 2007. Sraffa's Correspondence Relating to the Publication of *Production of Commodities by Means of Commodities*: Some Selected Material, paper presented at the annual meeting of the European Society for the History of Economic Thought, Strasbourg
- Harrod, R. F. 1961. Review of Production of Commodities by Means of Commodities. Prelude to a Critique of Economic Theory, The Economic Journal, vol. LXXI, 783-7
- Hicks, J. 1985. Sraffa and Ricardo: a critical view, pp. 305–19 in Caravale, G. A. (ed.), *The Legacy of Ricardo*, New York, Basil Blackwell
- Pasinetti, L. 2001. Continuity and change in Sraffa's thought: an archival excurses, pp. 139–56 in Cozzi, T. and Marchionatti, R. (eds), *Piero Sraffa's Political Economy: A Centenary Estimate*, London, Routledge
- Robinson, J. 1961. Prelude to a critique of economic theory, Oxford Economic Papers, vol. 13, 53-8

Roncaglia, A. 1978. Sraffa and the Theory of Prices, Chichester, John Wiley & Sons

Roncaglia, A. 2000. Piero Sraffa: His Life, Thought and Cultural Heritage, London, Routledge

- Sen, A. 2003. Sraffa, Wittgenstein, and Gramsci, *Journal of Economic Literature*, vol. 41, 1240–55 Sinha, A. 2010A. *Theories of Value from Adam Smith to Piero Sraffa*, London, Routledge
- Sinha, A. 2010B. A note on Ricardo's invariable measure of value, *Cahiers d'économie politique*, no. 58, 133–44
- Sinha, A. and Dupertuis, M.-S. 2009. Sraffa's system: equal rate of profits and the notion of centre of gravitation, *Journal of Economic Behavior and Organization*, vol. 71, 495–501
- Sraffa, P. 1951. Introduction, The Works and Correspondence of David Ricardo Vol. 1, Cambridge, UK, Cambridge University Press
- Sraffa, P. 1960. Production of Commodities by Means of Commodities, Cambridge, UK, Cambridge University Press
- Sraffa, P. 1962. Production of commodities. A comment, *The Economic Journal*, vol. LXXII, 477-9

Sraffa, P. Various years. Piero Sraffa Papers, Wren Library, Trinity College, Cambridge